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HEAD LOSS COMPUTATION IN A DIVERGENT CIRCULAR PIPE

CALCUL DE LA PERTE DE CHARGE DANS UNE CONDUITE CIRCULAIRE DIVERGENTE

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ABSTRACT

The head loss in the hydraulic systems falls into two categories, namely major and minor head losses. Major head losses are due to friction while minor head losses are due to components as valves, bends, the abrupt or gradual variation of a section of pipe and many other singularities. The term "minor losses" is a misnomer because, in most practical cases, they are not minor but on the contrary, they represent a significant part of the total head loss. Minor loss can be significant compared to the major loss. As a general rule, major head losses are evaluated using the Darcy-Weisbach relationship while minor head losses are determined using generally an empirical relation. Both relationships are related to kinetic energy which means that head loss is related to the square of the velocity.

In the present study, a rigorous development is proposed for the calculation of major losses along a divergent circular pipe. In the literature, there is no method for calculating friction losses in a divergent circular pipe. In the calculations, these losses are often neglected by the fact that they are insignificant. We will see, through a numerical practical example, that this is not always the case. The theoretical development is based on the integration of the Darcy-Weisbach equation along the divergent pipe. The determination of the exact relationship of the hydraulic diameter was necessary for the calculation of the friction factor according to Colebrook-White relation.

Keywords: Divergent pipe, circular pipe, head loss, major losses, minor losses.

RESUME

Les pertes de charge dans les systèmes hydrauliques se répartissent en deux catégories, à savoir les pertes de charge linéaires et les pertes de charge singulières. Les pertes de charge linéaires sont dues au frottement, tandis que les pertes de charge singulières sont dues à des composants tels que des vannes, des coudes, la variation abrupte ou progressive d'une section de tuyau et de nombreuses autres singularités. Le terme "pertes mineures" pour désigner les pertes de charge singulières est impropre dans la mesure où, dans la plupart des cas, elles ne sont pas mineures, mais au contraire, elles représentent une partie importante de la perte totale. Une perte de charge mineure peut être importante par rapport à une perte de charge majeure ou par frottement. En règle générale, les pertes de charge linéaires sont évaluées à l'aide de la relation de Darcy-Weisbach, tandis que les pertes de charge singulières sont déterminées à l'aide d'une relation empirique. Les deux relations sont liées à l'énergie cinétique, ce qui signifie que la perte de charge est proportionnelle au carré de la vitesse.

Dans la présente étude, un développement rigoureux est proposé pour le calcul des pertes de charge linéaires le long d'une conduite circulaire divergente. Dans la littérature, il n'existe pas de méthode permettant de calculer les pertes par frottement dans un tuyau circulaire divergent. Dans les calculs, ces pertes sont souvent négligées par le fait qu'elles sont insignifiantes. Nous verrons, à l'aide d'un exemple pratique numérique, que ce n'est pas toujours le cas. Le développement théorique est basé sur l'intégration de l'équation de Darcy-Weisbach le long de la conduite divergente. La détermination de la relation exacte du diamètre hydraulique était nécessaire pour le calcul du facteur de frottement selon la relation Colebrook-White.

Mots clés : Conduite divergente, conduite circulaire, perte de charge, pertes de charge singulières, pertes de charge linéaires.

INTRODUCTION

The head loss of a piping system is divided into two main categories, namely major losses associated with energy loss per length of pipe or friction head loss, and minor losses associated with bends, fittings, valves, gradual or abrupt change of the section and other system structures. Friction loss is caused by resistance to flowing water caused by the pipe walls. As such, these losses are inversely proportional to the diameter of the pipe (Paraschivoiu and Prud'homme, 2003). In fluid flow, friction loss (or skin friction) is the loss of pressure or head that occurs in pipe or duct flow due to the effect of the fluid's viscosity near the surface of the pipe or duct. The head loss is related to the square of the velocity (Rouse, 1946; Carlier, 1980) so the increase in loss is very quick. When the inside diameter is made larger, the flow area increases and the velocity of the liquid at a given flow rate is reduced. When the velocity is reduced there is lower head loss due to friction in the pipe. In fluid dynamics, the Darcy–Weisbach (Colebrook and White, 1937; Crowe et al., 2005; Afzal, 2007) equation is a semi-analytical equation, which relates the head loss, or pressure drop, due to friction along a given length of pipe to the average velocity of the fluid flow for an incompressible fluid. The equation is named after Henry Darcy and Julius Weisbach. In fluid dynamics, the Darcy–Weisbach equation is a phenomenological equation valid for fully developed, steady, incompressible single-phase flow (Rouse, 1946).

The Darcy friction factor is also known as the Darcy–Weisbach friction factor, resistance coefficient or simply friction factor. The phenomenological Colebrook–White equation (or Colebrook equation) expresses the Darcy friction factor f as a function of Reynolds number R and pipe relative roughness ε / D_h, fitting the data of experimental studies of turbulent flow in smooth and rough pipes (Colebrook, 1939). The equation can be used to (iteratively) solve the Darcy–Weisbach friction factor f.

Minor losses in pipe flow are a major part of calculating the flow, pressure, or energy reduction in piping systems. “Minor losses” is a misnomer because in many cases these losses are more important than the losses due to pipe friction. It will be shown in a practical example that the major losses in a draft tube are negligible compared to the minor head losses. They represent only a very small fraction of the total head loss. This one is equal to the sum of major losses and minor losses. For all minor losses in turbulent flow, the head loss varies as the square of the velocity. Thus a convenient method of expressing the minor losses in flow is using a minor loss coefficient k. Values of the minor loss coefficient k for typical situations and fittings are found in standard handbooks (Morel and Laborde, 1994; Idel'cik, 1986). These values are generally obtained experimentally. The minor loss coefficient k values ranges from 0 and upwards. For k = 0 the minor loss is zero and for k = 1 the minor loss is equal to the dynamic pressure or head. Major and minor losses in pipes are major contributing factors.

This technical note develops a rigorous method for the calculation of the major losses along a conical pipe. The study is based on known hydraulic relationships such as the Darcy-Weisbach relationship and the Colebrook-White relation for the calculation of the friction factor. This relationship requires in particular knowledge of the hydraulic diameter whose expression is developed in this study. In a numerical example, the major losses are calculated and compared to the minor losses and even to the total head loss. It appears that the major losses are not negligible although the minor losses are greater.

HYDRAULIC PARAMETERS

Wetted area A

Figure 1 shows schematically a conical shaped circular pipe. The inlet and outlet diameters are d_0 and d_1 respectively. The length of the pipe is L and the diameter at a distance x from the cone inlet is d_x. The inclination angle of the pipe wall with the horizontal is β, which means that the opening of the cone is 2β.

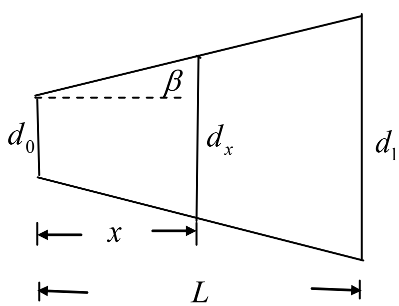

Figure 1 : Schematic representation of the cone-shaped pipe.

One can write:

$$d_x = d_0 + 2x\,tg\beta \quad (1)$$

$$A(x) = \frac{\pi d_x^2}{4} = \frac{\pi}{4}\left(d_0 + 2x\,tg\beta\right)^2 = \frac{\pi}{4}\left(d_0^2 + 4d_0\,x\,tg\beta + 4x^2\,tg^2\beta\right) \quad (2)$$

With $\alpha = 2tg\beta$

$$A\left(x\right)=\frac{\pi}{4}\left(d_0^2+2\alpha\,x\,d_0+\alpha^2\,x^2\right) \tag{3}$$

The wetted area *A* can be written as:

$$A=\frac{1}{L}\int_0^L A(x)dx \tag{4}$$

Thus:

$$A=\frac{1}{L}\int_0^L \frac{\pi}{4}\left(d_0^2+2\,\alpha\,x\,d_0+\alpha^2\,x^2\right)dx \tag{5}$$

$$A=\frac{\pi}{4L}\left[d_0^2\,L+\alpha\,L^2\,d_0+\frac{1}{3}\alpha^2\,L^3\right] \tag{6}$$

$$A=\frac{\pi}{4}\left[d_0^2+\alpha\,L\,d_0+\frac{1}{3}\alpha^2\,L^2\right] \tag{7}$$

From figure 1, the following relationship can be deduced :

$$d_1=d_0+\alpha L$$

So:

$$A=\frac{\pi}{4}\left[d_0^2+d_0\left(d_1-d_0\right)+\frac{1}{3}\left(d_1-d_0\right)^2\right] \tag{8}$$

$$=\frac{\pi}{4}\left[d_0^2+d_0d_1-d_0^2+\frac{1}{3}\left(d_1^2-2d_0d_1+d_0^2\right)\right] \tag{9}$$

$$\begin{aligned}A&=\frac{\pi}{4}\left[d_0^2+d_0d_1-d_0^2+\frac{1}{3}\left(d_1^2-2d_0d_1+d_0^2\right)\right]\\&=\frac{\pi}{4}\left[d_0^2+d_0d_1-d_0^2+\frac{d_1^2}{3}-\frac{2}{3}d_0d_1+\frac{1}{3}d_0^2\right]\end{aligned} \tag{10}$$

$$\begin{aligned}A&=\frac{\pi}{4}\left[d_0^2+d_0d_1-d_0^2+\frac{d_1^2}{3}-\frac{2}{3}d_0d_1+\frac{1}{3}d_0^2\right]\\&=\frac{\pi}{4}\left[\frac{1}{3}d_0d_1+\frac{d_1^2}{3}+\frac{1}{3}d_0^2\right]\end{aligned} \tag{11}$$

$$A = \frac{\pi}{4}\left[\frac{1}{3}d_0 d_1 + \frac{d_1^2}{3} + \frac{1}{3}d_0^2\right] = \frac{\pi}{12}\left(d_0^2 + d_1^2 + d_0 d_1\right) \quad (12)$$

Finally:

$$A = \frac{\pi}{12}\left(d_0^2 + d_1^2 + d_0 d_1\right) \quad (13)$$

It may be noted that the wetted area A is not calculated in the midsection, at L/2. At length L / 2, the wetted area is:

$$A_m = \frac{1}{2}\left[\frac{\pi\, d_0^2}{4} + \frac{\pi d_1^2}{4}\right] \quad (14)$$

The index "m" refers to the median section. The relation (14) can be written as:

$$A_m = \frac{\pi}{8}\left(d_0^2 + d_1^2\right) \quad (15)$$

In this case, What is the relative deviation between the exact mean wetted area and the arithmetical mean is:

$$\frac{\Delta A}{A_m} = \frac{A_m - A}{A_m} = 1 - \frac{A}{Am} \quad (16)$$

Thus :

$$\frac{\Delta A}{A_m} = 1 - \frac{\frac{\pi}{12}\left(d_0^2 + d_1^2 + d_0 d_1\right)}{\frac{\pi}{8}\left(d_0^2 + d_1^2\right)} \quad (17)$$

Finally:

$$\frac{\Delta A}{A_m} = 1 - \frac{2}{3}\frac{\left(d_0^2 + d_1^2 + d_0 d_1\right)}{\left(d_0^2 + d_1^2\right)} \quad (18)$$

EXAMPLE 1

Let us assume $d_0 = 1m$ and $d_1 = 2.5m$

$$\frac{\Delta A}{A_m} = 1 - \frac{2}{3}\frac{\left(1^2 + 2.5^2 + 1\times 2.5\right)}{\left(1^2 + 2.5^2\right)} = 0.10344828$$

If the calculation of the wetted area was taken in the middle section of the pipe, an error of about 10.35% would have been made in the calculation of *A*.

Curve of the relative deviation between the wetted areas

$$\frac{\Delta A}{A_m} = 1 - \frac{2}{3}\frac{\left(d_0^2 + d_1^2 + d_0 d_1\right)}{\left(d_0^2 + d_1^2\right)} = 1 - \frac{2}{3}\frac{d_1^2\left[\left(d_0/d_1\right)^2 + d_0/d_1 + 1\right]}{d_1^2\left[\left(d_0/d_1\right)^2 + 1\right]} \tag{19}$$

Thus:

$$\frac{\Delta A}{A_m} = 1 - \frac{2}{3}\frac{\left[\left(d_0/d_1\right)^2 + d_0/d_1 + 1\right]}{\left[\left(d_0/d_1\right)^2 + 1\right]} \tag{20}$$

$\frac{\Delta A}{A_m}\%$ given by the relationship (20) has been plotted in Figure 2 as a function of the values of d_0 / d_1.

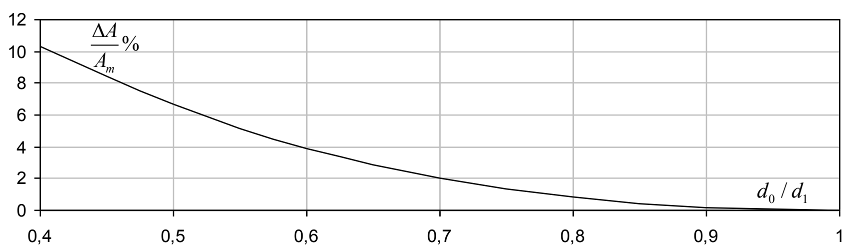

Figure 2 : Variation of $\frac{\Delta A}{A_m}\%$ **according to** d_0 / d_1

Figure 2 shows that the relative deviation $\frac{\Delta A}{A_m}\%$ decreases as the increase of d_0 / d_1. The relative deviation is zero for $d_0 / d_1 = 1$, corresponding to the circular pipe of constant diameter.

Wetted perimeter *P*

The mean wetted perimeter *P* can be expressed as:

$$P = \frac{1}{L}\int_0^L P(x)\,dx \tag{21}$$

With :

$$P(x) = \pi\, d_x = \pi\left(d_0 + 2\,x\,tg\beta\right) = \pi\left(d_0 + \alpha\, x\right) \tag{22}$$

$$P = \frac{1}{L}\int_0^L \pi\left(d_0 + \alpha\, x\right)dx$$

$$P = \frac{1}{L}\left(\pi\, d_0\, L + \frac{\pi}{2}\alpha\, L^2\right) \tag{23}$$

Thus :

$$P = \frac{\pi}{2}\left(\alpha\, L + 2\, d_0\right) \tag{24}$$

Knowing that $\alpha\, L = d_1 - d_0$, one can write :

$$P = \frac{\pi}{2}\left(d_0 + d_1\right) \tag{25}$$

It can thus be noted that the mean wetted perimeter *P* is equal to the mid-length perimeter. For the particular case of the circular pipe of constant diameter, one writes $d_0 = d_1 = d$. Thus:

$$P = \pi\, d \tag{26}$$

Hydraulic diameter D_h

The hydraulic diameter is defined as:

$$D_h = 4\frac{A}{P} \tag{27}$$

Inserting Eqs. (13) and (25) in (27), leads to :

$$D_h = 4\frac{\frac{\pi}{12}\left(d_0^2 + d_1^2 + d_0 d_1\right)}{\frac{\pi}{2}\left(d_0 + d_1\right)} \tag{28}$$

Thus:

$$D_h = \frac{2}{3}\frac{\left(d_0\, d_1 + d_1^2 + d_0^2\right)}{\left(d_0 + d_1\right)} \tag{29}$$

For the particular case of the circular pipe of constant diameter, corresponding to $d_0 = d_1 = d$, Eq. (29) gives:

$$D_h = \frac{2}{3}\frac{\left(d_0\, d_1 + d_1^2 + d_0^2\right)}{\left(d_0 + d_1\right)} = \frac{2}{3}\frac{\left(d^2 + d^2 + d^2\right)}{\left(2d\right)} = d \tag{30}$$

MAJOR HEAD LOSSES CALCULATION

The major head losses due to friction assumed as Δh_f is written as:

$$\Delta h_f\Big|_x = \int_0^x J\left(x\right)dx \tag{31}$$

Where J(x) is the major head loss gradient at distance x. The index «f» refers to friction. J is given by the well known Darcy-Weisbach relationship. That is:

$$J(x) = \frac{f}{D_h}\frac{V^2(x)}{2g} \tag{32}$$

f is the Darcy-Weisbach friction factor, *V*(x) is the flow velocity at the location *x* and g is the acceleration due to gravity. It is well known that $V = Q/A$, where *Q* is the discharge. It then comes as:

$$J(x) = \frac{f}{D_h}\frac{Q^2}{2gA^2(x)} \tag{33}$$

Inserting Eq. (27) in (33), one can write :

$$J(x) = \frac{f}{8g}\frac{Q^2}{A^3(x)}P(x) \tag{34}$$

The friction factor f is given by the Colebrook-White relationship as :

$$f = \left[-2\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{R\sqrt{f}}\right)\right]^{-2} \tag{35}$$

In this relationship, ε is the absolute roughness which characterizes the state of the inner wall of the pipe and *R* is the Reynolds number defined as:

$$R = \frac{4Q}{P\nu} \tag{36}$$

ν is the kinematic viscosity of the flowing liquid.

Taking into account the above, the minor head losses Δh_f can be written as:

$$\Delta h_f = \frac{f}{8g}\frac{Q^2}{A^3}PL \tag{37}$$

Δh_f can be expressed in the case of the cone-shaped pipe as:

$$\Delta h_f = \int_0^L \frac{f}{d(x)}\frac{Q^2}{2gA^2(x)}dx \tag{38}$$

If the hydraulic diameter $D_h(x) = 4A(x)/P(x)$ had been introduced instead of *d*(x), the same result would be obtained because, in any section of the conical pipe, the hydraulic diameter is equal to the geometrical diameter *d* since the section is circular.

Eq. (38) becomes:

$$\Delta h_f = \frac{f\,Q^2}{2g}\int_0^L \frac{dx}{A^2(x)d(x)} \tag{39}$$

$$\int_0^L \frac{dx}{A^2(x)d(x)} = \int_0^L \frac{dx}{\left(\frac{\pi}{4}d_x^2\right)^2 d(x)} \tag{10}$$

Knowing that [Eq.(1)]:

$$d_x = d_0 + 2\,x\,tg\beta = d_0 + \alpha x$$

So:

$$\int_0^L \frac{dx}{A^2(x)d(x)} = \int_0^L \frac{dx}{\left(\frac{\pi}{4}d_x^2\right)^2 d(x)} = \int_0^L \frac{dx}{\left[\frac{\pi}{4}\left(d_0+\alpha x\right)^2\right]^2 \left(d_0+\alpha x\right)}$$
$$= \int_0^L \frac{16\,dx}{\pi^2\left(d_0+\alpha x\right)^5} \tag{41}$$

Adopt the following change of variables:

$$u = d_0 + \alpha x \tag{42}$$

Thus:

$$du = \alpha\,dx \tag{43}$$

As a result:

$$\int_0^L \frac{16\,dx}{\pi^2\left(d_0+\alpha x\right)^5} = \int_0^L \frac{16\,du}{\alpha\pi^2 u^5} = \frac{16}{\pi^2}\int_0^L \frac{du}{\alpha\,u^5} \tag{44}$$

Or:

$$\frac{16}{\pi^2}\int_0^L \frac{du}{\alpha\,u^5} = \frac{16}{\pi^2}\left(-\frac{1}{4\alpha\,u^4}\right) = -\frac{4}{\alpha\,\pi^2\left(d_0+\alpha x\right)^4} \tag{45}$$

Thus:

$$\int_0^L \frac{16\,dx}{\pi^2\left(d_0+\alpha x\right)^5} = \left[-\frac{4}{\alpha\,\pi^2\left(d_0+\alpha x\right)^4}\right]_0^L = \frac{4}{\alpha\,\pi^2 d_0^4} - \frac{4}{\alpha\,\pi^2\left(d_0+\alpha L\right)^4} \quad (46)$$

Let us assume:

$$\Phi = \int_0^L \frac{16\,dx}{\pi^2\left(d_0+\alpha x\right)^5} \quad (47)$$

As a result:

$$\Phi = \frac{4}{\alpha\,\pi^2 d_0^4} - \frac{4}{\alpha\,\pi^2\left(d_0+\alpha L\right)^4} \quad (48)$$

Or:

$$\Phi = \frac{4}{\alpha\,\pi^2}\left[\frac{1}{d_0^4} - \frac{1}{\left(d_0+\alpha L\right)^4}\right] \quad (49)$$

Knowing that:

$$\alpha\,L = d_1 - d_0$$

Thus :

$$\Phi = \frac{4\,L}{\pi^2\left(d_1-d_0\right)}\left[\frac{1}{d_0^4} - \frac{1}{\left(d_0+d_1-d_0\right)^4}\right] \quad (50)$$

Finally:

$$\Phi = \frac{4\,L}{\pi^2\left(d_1-d_0\right)}\left(\frac{1}{d_0^4} - \frac{1}{d_1^4}\right) \quad (51)$$

Eq.(39) can be then written as:

$$\Delta h_f = \frac{f\,Q^2}{2g}\Phi \quad (52)$$

Combining Eqs (51) and (52), yields:

$$\Delta h_f = \frac{f\,Q^2}{2g}\frac{4\,L}{\pi^2\left(d_1 - d_0\right)}\left(\frac{1}{d_0^4} - \frac{1}{d_1^4}\right) \tag{53}$$

After simplifications, it comes that:

$$\Delta h_f = \frac{2f\,Q^2\,L}{g\,\pi^2\left(d_1 - d_0\right)}\left(\frac{1}{d_0^4} - \frac{1}{d_1^4}\right) \tag{54}$$

For the pressurized circular pipe of constant diameter, corresponding to $d_0 = d_1 = d$, Eq.(54) results in an indeterminacy. But, we can continue to develop this last relationship by writing that:

$$\begin{aligned}\left(\frac{1}{d_0^4} - \frac{1}{d_1^4}\right) &= \frac{d_1^4 - d_0^4}{d_0^4 d_1^4} = \frac{\left(d_1^2 - d_0^2\right)\left(d_1^2 + d_0^2\right)}{d_0^4 d_1^4} \\ &= \frac{\left(d_1 - d_0\right)\left(d_1 + d_0\right)\left(d_1^2 + d_0^2\right)}{d_0^4 d_1^4}\end{aligned} \tag{55}$$

Thus:

$$\Delta h_f = \frac{2f\,Q^2\,L}{g\,\pi^2\left(d_1 - d_0\right)}\frac{\left(d_1 - d_0\right)\left(d_1 + d_0\right)\left(d_1^2 + d_0^2\right)}{d_0^4 d_1^4} \tag{56}$$

After simplifications, one can write:

$$\Delta h_f = \frac{2f\,Q^2\,L}{g\,\pi^2}\frac{\left(d_1 + d_0\right)\left(d_1^2 + d_0^2\right)}{d_0^4 d_1^4} \tag{57}$$

For $d_0 = d_1 = d$, Eq.(57) leads to:

$$\Delta h_f = \frac{2f\,Q^2\,L}{g\,\pi^2}\frac{\left(2d\right)\left(2d^2\right)}{d^8} = \frac{8f\,Q^2\,L}{g\,\pi^2 d^5} \tag{58}$$

Thus:

$$\Delta h_f = \frac{8f\,Q^2\,L}{g\,\pi^2 d^5} \tag{59}$$

MINOR HEAD LOSSES COMPUTATION

The pipe represented in figure 1 causes minor head losses Δh_m such that:

$$\Delta h_m = k\frac{V_0^2}{2g} \tag{60}$$

Where *k* is the minor loss coefficient given by the following relationship (Morel and Laborde, 1994):

$$k = b\left(\frac{d_1^2}{d_0^2} - 1\right)^2 \tag{61}$$

The parameter *b* is a coefficient which depends on the opening angle of the conical pipe and whose values are indicated in table 1. Knowing that $V_0 = Q / A_0 = 4Q / (\pi d_0^2)$, one can write:

$$\Delta h_m = k\frac{(4Q)^2}{2g\left(\pi d_0^2\right)^2} = k\frac{8Q^2}{g\pi^2 d_0^4} \tag{62}$$

Thus:

$$\Delta h_m = k\frac{8Q^2}{g\pi^2 d_0^4} \tag{63}$$

Combining Eqs.(61) and (63), results in:

$$\Delta h_m = b\left(\frac{d_1^2}{d_0^2} - 1\right)^2 \frac{8Q^2}{g\pi^2 d_0^4} \tag{64}$$

Table 1: Values of the parameter b as a function of the opening angle of the cone-pipe according to Morel and Laborde (1994)

2β	5°	6°	7°	8°	10°
b	0,049	0,062	0,075	0,088	0,119

2β	16°	18°	20°	30°	40°
b	0,245	0,307	0,389	0,80	0,90

TOTAL HEAD LOSS RELATIONSHIP

The total head loss Δh_T corresponds to the sum of the major head losses Δh_f and the minor head losses Δh_s, that is:

$$\Delta h_T = \Delta h_f + \Delta h_m \tag{65}$$

Taking into account Eqs.(57) and (64), Eq.(65) becomes:

$$\Delta h_T = \frac{2 f Q^2 L}{g \pi^2} \frac{\left(d_1 + d_0\right)\left(d_1^2 + d_0^2\right)}{d_0^4 d_1^4} + b\left(\frac{d_1^2}{d_0^2} - 1\right)^2 \frac{8Q^2}{g\pi^2 d_0^4} \tag{66}$$

Eq. (66) is reduced to:

$$\Delta h_T = \frac{2Q^2}{g \pi^2 d_0^4}\left[\frac{\left(d_1 + d_0\right)\left(d_1^2 + d_0^2\right)}{d_1^4} f L + 4b\left(\frac{d_1^2}{d_0^2} - 1\right)^2\right] \tag{67}$$

EXAMPLE 2

Consider the pipe shown in Figure 1 with the following data:

$Q = 0.08 m^3/s$; $d_0 = 0.2 m$; $d_1 = 0.4 m$; $\varepsilon = 0.001 m$; $\nu = 10^{-6} m^2/s$; $2\beta = 5°$

This results in:

$$tg\beta = tg(2.5°) = 0.043660943$$
$$\alpha = 2tg\beta = 2 \times 0.043660943 = 0.087321886$$

The length of the pipe is, therefore:

$$L = \left(d_1 - d_0\right)/\alpha = (0.4 - 0.2)/0.087321886 = 2.290376555 m \approx 2.3 m$$

According to Eq.(13), the mean wetted area is :

$$A = \frac{\pi}{12}\left(d_0^2 + d_1^2 + d_0 d_1\right) = \frac{\pi}{12}\left(0.2^2 + 0.4^2 + 0.2 \times 0.4\right) = 0.073303829 m^2$$

The mean wetted perimeter is given by Eq.(23) as:

$$P=\frac{\pi}{2}\left(d_0+d_1\right)=\frac{\pi}{2}\left(0.2+0.4\right)=0.942477796\,m$$

Applying Eq.(29), the hydraulic diameter is:

$$D_h=\frac{2}{3}\frac{\left(d_0\,d_1+d_1^2+d_0^2\right)}{\left(d_0+d_1\right)}=\frac{2}{3}\times\frac{\left(0.2\times0.4+0.4^2+0.2^2\right)}{\left(0.2+0.4\right)}=0.311111111m$$

The hydraulic diameter could have been calculated simply by the relation $D_h=4A/P$, which would have led to the same result.

As a result, the relative roughness is:

$$\varepsilon/D_h=0.001/0.311111111=0.003214286$$

According to Eq.(36), Reynolds number is :

$$R=\frac{4Q}{P\nu}=\frac{4\times0.08}{0.942477796\times10^{-6}}=339530.5453$$

As a result, the friction factor f according to Colebrook-White [Eq.(35] is:

$$f=0.0270656$$

The friction factor f can also be calculated explicitly by approximate relationships that are available in the literature (Achour, 2006: Zeghadnia et al., 2019)

According to Eq.(58), the major head losses Δh_f are as:

$$\Delta h_f=\frac{2f\,Q^2\,L}{g\,\pi^2}\frac{\left(d_1+d_0\right)\left(d_1^2+d_0^2\right)}{d_0^4d_1^4}$$

$$=\frac{2\times0.0270656\times0.8^2\times2.290376555}{9.81\times\pi^2}$$

$$\times\frac{\left(0.2+0.4\right)\times\left(0.4^2+0.2^2\right)}{0.2^4\times0.4^4}$$

Thus:

$$\Delta h_f = 0.024009718\,m \approx 2.4\,cm$$

It may seem that the major head losses are low, but we will see later that they are not negligible compared to the minor head losses. The more the angle 2β increases, the more Δh_f decreases.

Let us compute the minor head losses Δh_m according to Eq.(64):

$$\Delta h_m = b\left(\frac{d_1^2}{d_0^2} - 1\right)^2 \frac{8Q^2}{g\pi^2 d_0^4}$$

The value of the parameter *b* is given by Table 1 for the value of the angle 2β. For 2β=5°, the table gives b=0,049. Thus, the minor head losses are:

$$\Delta h_m = b\left(\frac{d_1^2}{d_0^2} - 1\right)^2 \frac{8Q^2}{g\pi^2 d_0^4} = 0.049 \times \left(\frac{0,4^2}{0.2^2} - 1\right)^2$$

$$\times \frac{8 \times 0.08^2}{9.81 \times \pi^2 \times 0.2^4} = 0.145753776\,m$$

Approximately:

$$\Delta h_m \approx 14.6\,cm$$

We can therefore notice that the minor head losses Δh_m are greater than the major head losses Δh_f. But, the latter represents about 16.5% of Δh_m and can not be neglected.

The total head loss Δh_T is then:

$$\Delta h_T = \Delta h_f + \Delta h_m = 0.024009718 + 0.145753776 = 0.169763494\,m$$

Thus:

$$\Delta h_T \approx 17\,cm$$

We can also notice that the major head losses Δh_f represent more than 14% of the total head loss Δh_T, which is not negligible.

EXAMPLE 3

This example is taken from a real case of a draft tube placed in a Kaplan turbine at the Gezhouba power plant, Hubei Province, People's Republic of China.

The data are as follows:

$Q = 825\,m^3/s$; $d_0 = 8.84\,m$; $d_1 = 12.2\,m$; $\varepsilon = 0.002\,m$; $\nu = 10^{-6}\,m^2/s$; $\beta = 11°26'$

Thus :

Transforming the angle β into degrees, either $\beta = 11°26' = 11.433333°$; or else $2\beta = 22,866666°$.

$$\alpha = 2tg\beta = 2 \times tg(11.433333) = 0.40448173$$

The length of the conical diffuser is therefore:

$$L = \left(d_1 - d_0\right)/\alpha = (12.2 - 8.84)/0.40448173 \approx 8.307\,m$$

The wetted area is:

$$A = \frac{\pi}{12}\left(d_0^2 + d_1^2 + d_0 d_1\right) = \frac{\pi}{12}\left(8.84^2 + 12.2^2 + 8.84 \times 12.2\right)$$
$$= 87.65923149\,m^2$$

The wetted perimeter is:

$$P = \frac{\pi}{2}\left(d_0 + d_1\right) = \frac{\pi}{2}\left(8.84 + 12.2\right) = 33.04955472\,m$$

The hydraulic diameter is:

$$D_h = \frac{2}{3}\frac{\left(d_0\,d_1 + d_1^2 + d_0^2\right)}{\left(d_0 + d_1\right)} = \frac{2}{3} \times \frac{\left(8.84 \times 12.2 + 8.84^2 + 12.2^2\right)}{\left(8.84 + 12.2\right)}$$
$$= 10.60942966\,m$$

As a result, the relative roughness is:

$$\varepsilon / D_h = 0.002/10.60942966 = 0.000188512$$

Reynolds number is:

$$R = \frac{4Q}{P\nu} = \frac{4 \times 825}{33.04955472 \times 10^{-6}} = 99850059.35$$

According to the Colebrook-White equation, friction factor f was obtained as:

$$f = 0.0135774$$

Major head losses Δh_f are then:

$$\Delta h_f = \frac{2 f Q^2 L}{g \pi^2} \frac{(d_1 + d_0)(d_1^2 + d_0^2)}{d_0^4 d_1^4}$$

$$= \frac{2 \times 0.0135774 \times 825^2 \times 8.307}{9.81 \times \pi^2} \times \frac{(12.2 + 8,84) \times (12.2^2 + 8.84^2)}{8.84^4 \times 12.2^4}$$

Thus:

$$\Delta h_f = 0.055979063\, m \approx 5.6\, cm$$

We can also notice that the major head losses are low. Let's calculate the minor head losses Δh_m :

$$\Delta h_m = b \left(\frac{d_1^2}{d_0^2} - 1 \right)^2 \frac{8Q^2}{g \pi^2 d_0^4}$$

The value of the parameter b is given by table1 for the value of the angle 2β. For $2\beta = 22.866666°$, table 1 indicates $b = 0.50681973$, value obtained by linear interpolation. Thus, the minor head losses Δh_m are:

$$\Delta h_m = b \left(\frac{d_1^2}{d_0^2} - 1 \right)^2 \frac{8Q^2}{g \pi^2 d_0^4} = 0.50681973 \times \left(\frac{6^2}{4^2} - 1 \right)^2 \times \frac{8 \times 825^2}{9.81 \times \pi^2 \times 8.84^4}$$
$$= 3.819746575\, m$$

Approximately:

$$\Delta h_m \approx 3.82\, m$$

We can thus notice that the minor head losses are very important compared to the major head losses. In fact, the major head losses Δh_f represent about 1.5% only of the minor head losses Δh_m.

The total head loss Δh_T is then:

$$\Delta h_T = \Delta h_f + \Delta h_m = 0.055979063 + 3.819746575 = 3.875725638\,m$$

Approximately:

$$\Delta h_T \approx 3.88\,m$$

Finally, major head losses Δh_f represent about 1.44% of total head loss, which is negligible. Several applications we have done have shown this fact. We can conclude in practice, in the case of the draft tube, that the major head losses Δh_f are negligible.

CONCLUSIONS

The purpose of the study was to calculate the major and minor head losses in a divergent circular pipe. The theoretical development was based on the integration of the Darcy-Weisbach relationship along the pipe. All parameters of the flow have been represented in this relation. It was necessary to establish the exact relationship of the hydraulic diameter to calculate the friction coefficient according to the Colebrook-White relation. It has been found that the hydraulic diameter should not be calculated in the middle section of the pipe unlike the wet perimeter which is evaluated in the middle of the pipe. Through calculation examples, it has been shown that the friction losses must not always be neglected in such a pipe. On the other hand, in the case of draft tubes, these losses are almost insignificant. Several examples we have treated have shown this fact.

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