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CONVECTION HEAT TRANSFER AND MAGNETOHYDRODYNAMICS STABILITY OF SWIRLING FLOWS IN CYLINDRICAL CONTAINER

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ABSTRACT

In the present work, a study of the mixed convection flow of low Prandtl number fluid ($\mathrm{Pr} = 0.015$) confined in a cylindrical container having an aspect ratio equal to 2, with and without magnetic fields, has been considered. The finite volumes method has been used to resolve the equations of continuity, momentum (or Navier-Stokes), energy and electric potential. In the absence of magnetic field, the numerical results obtained show the appearance of oscillatory instabilities for the values of the critical Reynolds number $\mathrm{Re}_{cr} = 2575, 924, 802,$ and 606, corresponding respectively to the values of the Richardson number $Ri = 0$, 0.5, 1.0 and 2.0. However, in the presence of the vertical magnetic field, the fluid continues its stable flow until the values of Reynolds number greater than those predictable to have oscillatory instabilities. Stability diagrams have been established according to the numerical results of this investigation. These diagrams put in evidence the dependence of the critical Reynolds number and critical frequency of oscillations with the increase of the Hartmann number for various values of the Richardson number. In conclusion, the stabilizing technique of the mixed convection flows of fluids having low Prandtl number (semiconductors) by the application of an external magnetic field is practically reliable.

Keywords: *mixed convection, magnetic field, instabilities, heat transfer*

1. INTRODUCTION

The incompressible viscous fluid flow confined in a cylindrical enclosure induced by the rotation of one or more walls of the cylinder which contains the fluid was studied intensively and on several occasions during the last years. This type of flows can occur in many practical situations [1]: rotational viscosimeters, centrifugal machinery, pumping of liquid metals at high melting point, crystal growth from molten silicon in Czochralski crystal pullers [2], geophysical systems [3]... etc.

After this foreword, we expose some work available in the literature which treats the flow in question, with and without heat transfer by forced and mixed convection. Gelfgat et al.[4] presented a very detailed numerical study stable

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states and beginning of oscillatory instabilities of the incompressible Newtonian fluid flow confined in a vertical cylinder. Bessaïh et al.[2] carried a numerical study on rotating MHD laminar flow of a liquid metal contained in a cylindrical enclosure, having an aspect ratio equal to 1 and subjected to a vertical external magnetic field. A good agreement between the asymptotic and numerical results was obtained by the authors. They showed that we can control the primary flow by a good choice of the electric conductivity of the enclosure walls in question. Bessaïh et al.[3] carried out a numerical and analytical combined study of the same flow already mentioned in [2]. They showed the strong dependence of the flow and heat transfer structures with the magnetic field and the electric conductivity of the walls constituting the cylindrical enclosure.

The present work investigates numerically the determination of hydrodynamic and thermal instabilities which are created in a cylindrical chamber having an aspect ratio equal to 2, filled with a liquid metal and having a rotating top disk. This configuration (Fig.1) is subjected to a constant vertical magnetic field. We determine the critical value of the Reynolds number Re_{cr} for each value of Richardson number, $Ri = Gr/\mathrm{Re}^2 = 0$, 0.5, 1 and 2, and each value of Hartmann number $Ha = BR\sqrt{\pm / °³} = 0$, 5, 10, 20, 30, 40, 50, and 60.

2. GEOMETRY AND MATHEMATICAL MODEL

The geometry of the flow field analyzed in this study is illustrated in Fig. 1. The flow field driven by a rotating top wall, with an angular velocity Ω, is assumed to be axisymmetric. A liquid metal with a density ° , a kinematics viscosity ³ and an electrical conductivity ± , fills a cylinder of radius R and height H is submitted to an axial magnetic field B. The top end wall rotates with a constant angular velocity Ω. The bottom wall is kept at a local hot temperature T_h , the top rotating disk is maintained at a local cold temperature T_c $(T_c < T_h)$, and the sidewall is adiabatic.

The dimensionless equations describing the flow: the continuity, Navier Stokes, energy and potential equations, together with appropriate boundary conditions in the cylindrical coordinate system $(r, ¯, z)$ are

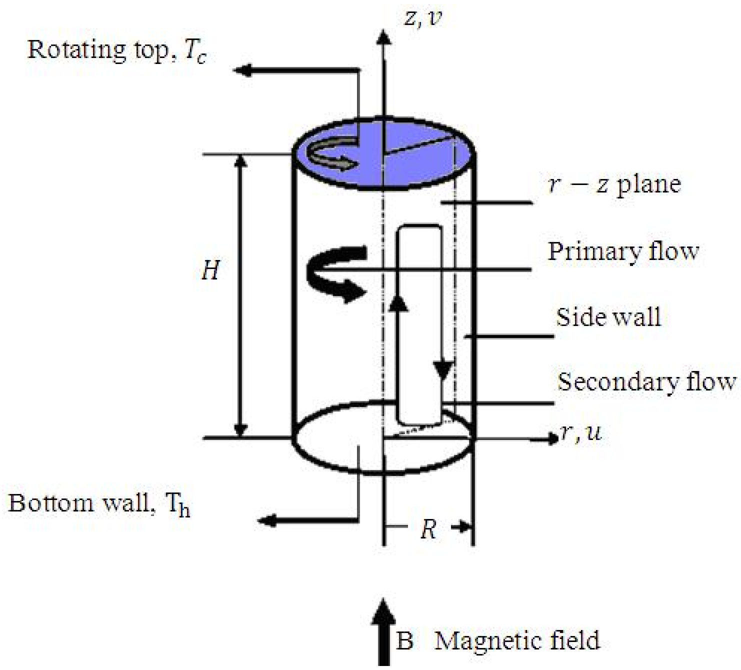

Fig.1. Geometry of the physical problem.

$$\frac{1}{r}\frac{\partial (ru)}{\partial r}+\frac{\partial v}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial ^2}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}-\frac{w^2}{r}=-\frac{\partial P}{\partial r}+\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right]+N\,F_{Lr} \tag{2}$$

$$\frac{\partial v}{\partial ^2}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=-\frac{\partial P}{\partial z}+\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+\frac{\partial^2 v}{\partial z^2}\right]+Ri\cdot\Theta \tag{3}$$

$$\frac{\partial w}{\partial ^2}+u\frac{\partial w}{\partial r}+v\frac{\partial w}{\partial z}-\frac{uw}{r}=\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial z^2}-\frac{w}{r^2}\right]+N\,F_{L-} \tag{4}$$

$$\frac{\partial \Theta}{\partial ^2}+u\frac{\partial \Theta}{\partial r}+v\frac{\partial \Theta}{\partial z}=\frac{1}{\mathrm{Re}\cdot\mathrm{Pr}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Theta}{\partial r}\right)+\frac{\partial^2 \Theta}{\partial z^2}\right] \tag{5}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right)+\frac{\partial^2 \Phi}{\partial z^2}=\frac{w}{r}+\frac{\partial w}{\partial r} \tag{6}$$

At $^2 = 0,\quad u=0, v=0,\ w=0,\ \Theta=0,\ \Phi=0 \qquad (0<r<1,\ 0<z<¥)$

For $ऱ > 0$,

$$u=0,\ \ \frac{\partial v}{\partial r}=0,\quad w=0,\ \ \frac{\partial \Theta}{\partial r}=0,\ \ \frac{\partial \Phi}{\partial r}=0 \qquad (r=0,\ \ 0\le z\le \gamma).$$

(7a)

$$u = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial \Theta}{\partial r} = 0, \quad \frac{\partial \Phi}{\partial r} = 0 \qquad (r = 1, \quad 0 \le z \le \gamma).$$

(7b)

$$u = 0, \quad v = 0, \quad w = r, \quad \Theta = 0, \quad \frac{\partial \Phi}{\partial z} = 0 \qquad (z = \gamma, \quad 0 \le r \le 1).$$

(7c)

$$u = 0, \quad v = 0, \quad w = 0, \quad \Theta = 1, \quad \frac{\partial \Phi}{\partial z} = 0 \qquad (z = 0, \quad 0 \le r \le 1).$$

(7d)

The governing equations were solved using a finite volume method (see, Patankar [4]). Scalar quantities (P, Θ, w, Φ) are stored in the centre of these volumes, whereas the vectorial quantities (u and v) are stored on the faces. For the discretisation of spatial terms, a second-order central difference scheme was used for the diffusion and convection parts of the equations (2-5), and the SIMPLER algorithm [4] was used to determine the pressure from continuity equation. The grid used has 80×160 nodes and was chosen after performing grid independency tests, since it is considered to have the best compromise between the computing time and the sufficient resolution in calculations. Calculations were carried out on a PC with 2.8 GHz CPU, thus, the average computing time for a typical case was approximately of 8 hours.

3. RESULTS AND DISCUSSION

3.1. Validation of the code

With an aim of allotting more confidence to the results of our numerical simulations, we have established some comparisons with the experimental investigation presented in the literature [5]. Comparison has been made for the axial distribution of the azimuthal velocity w at $r = 0.60$, with experimental measurements obtained by Michelson [5], which used the LDA technique to determine the azimuthal velocity w in a cylindrical cavity, whose higher disk is in rotation, for $Re = 1800$ and $\gamma = 1$ (Fig. 2). It is clear that the computed values can be seen to be in excellent agreement with measurements over the whole flow field.

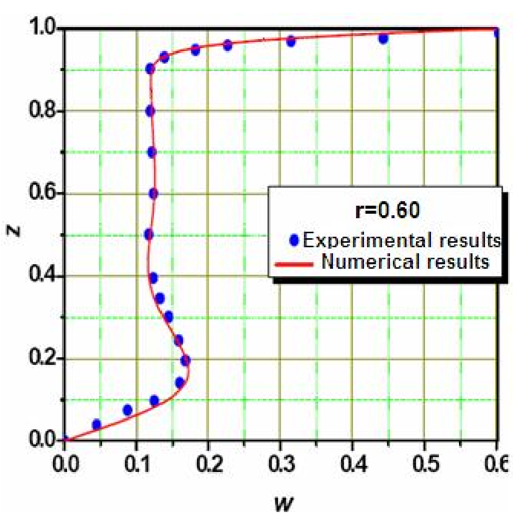

Fig. 2. Validation of the code with experimental measurements (Michelson [5]), for $Re = 1800, \gamma = 1$ and $Ha = 0$.

3.2. Solution with magnetic field ($Ha \neq 0$)

In the presence of the magnetic field B, the increase of Reynolds numbers Re, the flow beyond those known as critical will generate a junction of the flow towards the unstable mode (Fig.3), as well as a multiplicity of the frequencies of oscillations in the flow will take place, as mentioned in [6]. This is illustrated in figure 3b, which shows the prevalent frequencies of oscillation for some cases of the oscillatory flow. This spectral analysis is the result of the application of the fast Fourier transform of temporal evolutions of some parameters.

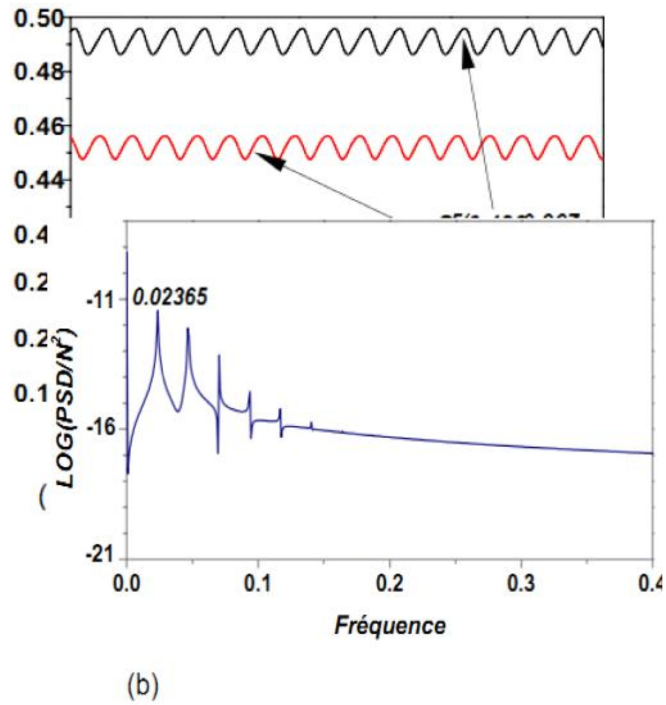

Fig. 3. Temporal evolutions for $\mathrm{Re}_{cr} = 961,\ Ri = 0.5$ and $Ha = 5$: (a) temperature Θ and (b) density of the spectrum of energy according to the frequency. Where S_5 (r=0.486, z=0.967), S_6 (r=0.90, z=0.967, and S_7(r=0.099, z=1.80) are the probes of recordings.

We present the temporal evolution during one period of the dimensionless radial velocity u for the case where $\mathrm{Re}_{cr} = 961,\ Ri = 0.5$ and $Ha = 5$, and we indicate the various moments noted by: ${}^{2}{}_{a},{}^{2}{}_{b},{}^{2}{}_{c},{}^{2}{}_{d},{}^{2}{}_{e},{}^{2}{}_{f},{}^{2}{}_{h},{}^{2}{}_{g}$ (fig. 4).

Figure 5 shows the dimensionless stream functions · . During the dimensionless times $(({}^{2}{}_{a},{}^{2}{}_{b},{}^{2}{}_{c}....)$,we notice the existence of a simple cell close to the side wall having a positive mass flow (in dotted lines). Also, this cell dilates and narrowed during time $({}^2{}_a, {}^2{}_b, {}^2{}_c ....)$, this process for one period of $1/F_{cr} = 1/0.02365 \approx 42.28$.

The application of a vertical magnetic field is recognized on the stability of the convective flows [2-4]. With regard to the dependence between the critical Reynolds number and the magnetic field intensity, the growth of Re_{cr} with the increase of Ha clearly seen in the stability diagram (Re_{cr} - Ha), (Fig. 6). This growth is monotonous except for the case for $Ri = 2.0$ and $Ha = 30$, where a slight reduction of Re_{cr}, (this case was also obtained by Gelfgat [6]). This growth is explained by the interaction of the vertical magnetic field on the mixed convection flow, this one is produced with the radial component of the velocity. Consequently, a stronger magnetic field is necessary to keep the stable flow for certain high values of the Reynolds number.

4. CONCLUSIONS

A numerical study of the mixed convection n a cylindrical enclosure filled with a liquid metal, subjected to a vertically magnetic field, has been made. The finite volumes method has been used to solve numerically the transport equations. Our numerical simulations have been presented for various values of the Hartmann (Ha = 0, 5, 10, 20, 30, 40, 50 and 60) and various values of the Richardson number (Ri = 0., 0.5, 1.0, and 2.0), in order to see their effects on the value of the critical Reynolds number, Re_{cr} and of the critical frequency of oscillation, F_{cr} in the presence of the vertical magnetic field, the fluid continues its stable flow up to the values of Reynolds number larger than those foreseeable to have oscillatory instabilities although the application of a magnetic field causes a remarkable change of the flow and heat transfer structures.

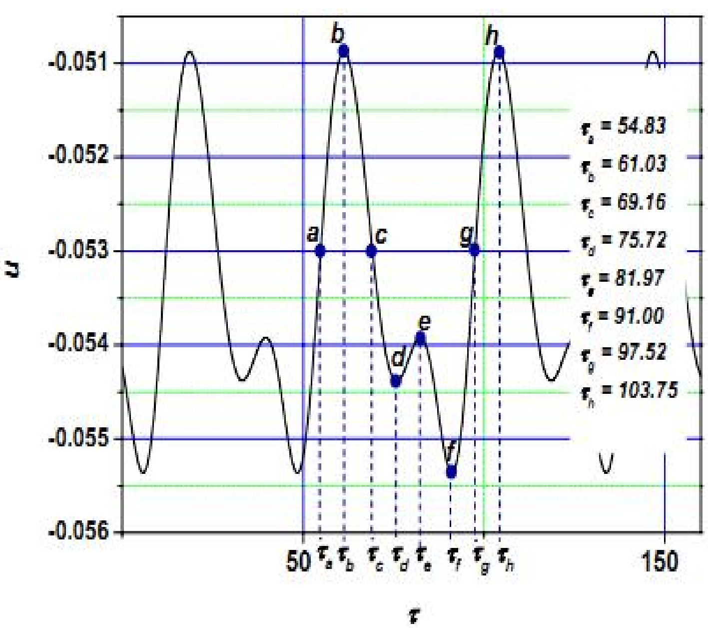

Fig. 4. Temporal evolution of u at point S_1 (0.099, 0.2) with various times ^{2} (a, b, c, d, e, f, g, h), for $\mathrm{Re}_{cr} = 961$, $Ri = 0.5$ and $Ha = 5$.

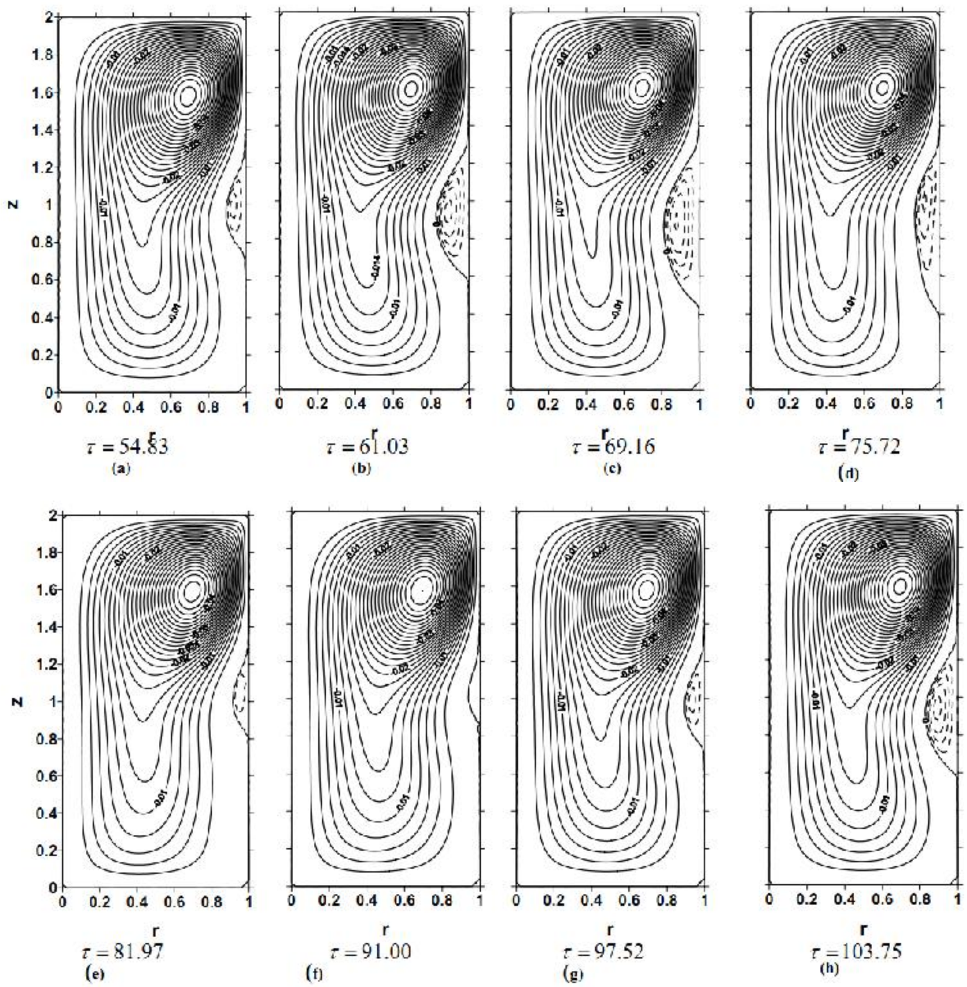

Fig. 5. Time history of the dimensionless streamlines · at various dimensionless times for $\mathrm{Re}_{cr} = 3504$, $Ri = 0.5$ (predominant forced convection), and $Ha = 60$.

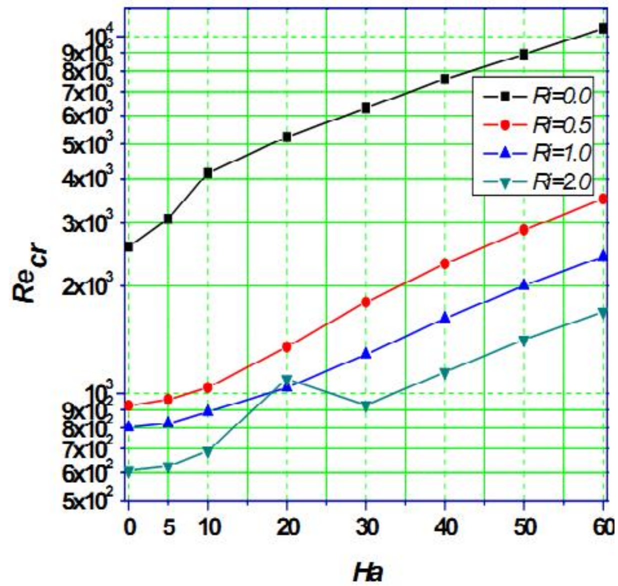

Fig. 6. Stability diagram (Re_{cr} - Ha).

NOMENCLATURE

B	Magnetic field, Tesla
F	Dimensionless Frequency
F_{Lr} , F_{L-}	Dimensionless Lorentz force in the radial and azimuthal directions, respectively
g	Gravitational acceleration, m.s^{-2}
H	Height of the cylinder, m
N	Interaction parameter
P	Dimensionless pressure
R	Radius of the cylinder, m
r, ¯ , z	Dimensionless radius, azimuthal and height
T_h ,T_c	Temperature of the hot and cold walls, K
u	Dimensionless radial velocity
v	Dimensionless axial velocity
w	Dimensionless azimuthal velocity

Greek Symbols

³	Kinematic viscosity ,m^2 /s
°	Density of the fluid , kg /m^3
±	Electrical conductivity , 1/ Ω m
	Thermal expansion coefficient, 1/K
Ω	Angular velocity, rad /s
Φ	Dimensionless electric potential
·	Dimensionless stream function
Θ	Dimensionless temperature

¥ Aspect ratio
² Dimensionless time

Non-dimensional Numbers

Gr Grashof number
Ha Hartmann number
Pr Prandtl number
Re Reynolds number
Ri Richardson number

5. REFERENCES

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