

## **AREAL (2-D) SIMULATION OF WATER FLOOD PROCESS IN UNITE WELL PATTERN**

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### **ABSTRACT**

In this project, the partial differential equations governing water flood in a unit well pattern were solved numerically and then the numerical solutions were used to construct the contour and surface plots of reservoir pressure and water saturation. The reservoir pressure and water saturation at a given location in the reservoir as a function of injection time were also simulated. The simulation results show that the reservoir pressure and water saturation gradually decreases with increasing the distance from the injection well. In addition, the reservoir pressure and water saturation gradually increase with increasing injection time at the location of (20 m, 20 m) in the reservoir. The water saturation gradually increases with increasing injection time at the production well.

**Keywords:** *Water flood, Simulation, Water saturation, Pressure*

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### **1. INTRODUCTION**

After the geological structure of a reservoir is studied, the sedimentary basin is analyzed, and reservoir is characterized in the respect of porosity and permeability, crude oil is mainly recovered through the drilling of oil well. Depending on the production life of a reservoir, oil recovery can be classified into three stages: primary, secondary, and tertiary [1-5]. The first stage of oil recovery is primary recovery, which comes from initially available natural energy in the reservoir. The natural energy includes natural water displacing oil downward into the well, expansion of the natural gas at the top of the reservoir and expansion of rock, expansion of initially dissolved gas in the crude oil, and gravity drainage resulting from the movement of oil within the reservoir from the upper to the lower parts where the wells are located [6-10].

The second stage of oil recovery is secondary recovery, which begins when the well pressure falls until there is not enough underground pressure to force the oil to move up to the surface. Secondary recovery works by providing water into the

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reservoir to maintain or increase reservoir pressure. Thus secondary recovery method is also known as water flood. Water flood is a widely used for secondary recovery method in the oil industry. During a typical water flood process, water is injected into the injection wells in a reservoir while the hydrocarbons in the reservoir are produced from designed production wells.

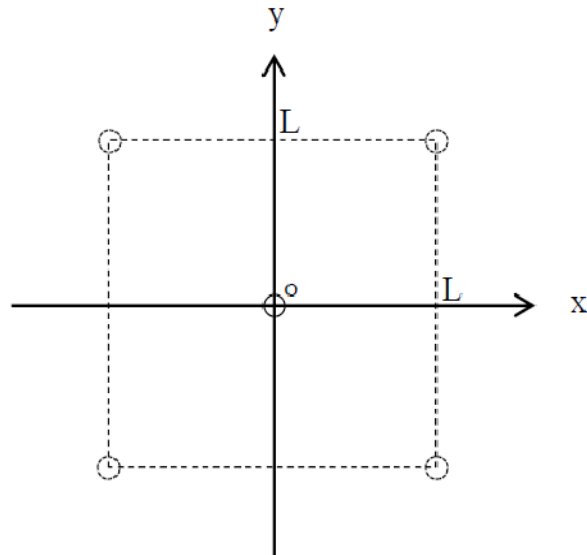
There are several important partial differential equations, which describe the reservoir pressure and water saturation profiles, govern a water flood process. The reservoir pressure and water saturation profiles during a water flood process can help us understand the process better and simulation is a good tool to study such profiles [11-15]. Thus in this project, the partial differential equations governing a water flood in a unit well pattern were solved numerically and then the numerical solutions were used to construct the contour and surface plots of reservoir pressure and water saturation. The reservoir pressure and water saturation at a given location in the reservoir as a function of injection time were also simulated.

## **2. MATHEMATICAL MODELS**

In this project, a 2-D water flood process is simulated. Specifically, in this project, a 2-D water flood process is considered in a square reservoir, where there are four water injection wells in the four corners of the reservoir and there is one fluid production well in the centre of the reservoir (Fig. 1). Since the reservoir is symmetric as to the production well, let only consider the domain ( $\Omega$ ) in the Quadrant I of Fig. 1. And the boundary of this domain consists of three parts: water injection boundary ( $\Gamma_{in}$ ), fluid production boundary ( $\Gamma_{out}$ ) and impermeable boundary ( $\Gamma_{imp}$ ).

Assumptions used in this project are the reservoir is a homogenous formation with constant permeability and porosity. The water flood process is considered as a two-phase incompressible flow of oil and water with constant viscosity of each. The fluid flow through the formation follows Darcy's Law. The process is at a constant temperature, and the capillary and gravitational effects are neglected [15].

These treatments are mostly based on physico-chemical processes and are sometimes Sir most specific economic, certain types of load or certain operating conditions although they are, in principle, able to handle all aromatic types' essences. They are: Crystallization, adsorption, distillation, azeotropic distillation, extractive distillation and solvent extraction [5, 7].



**Fig.1.**Schematic diagram for an areal (2-D) water flood process in unit well.

Pattern, where dashed cycle stands for water injection well and solid cycle stand for production well.

Under the above assumptions, the water flood process is governed by a nonlinear partial differential equation system with a pressure equation and a saturation equation [1, 3, 5, 8].

$$\begin{cases} \text{div}(-KM(S)\nabla P) = 0 \\ \phi \partial_t S + \text{div}(Vf(S)) = 0 \\ V = -KM(S)\nabla P \\ 0 \leq S \leq 1 \end{cases} \quad (1)$$

Where  $M(S) = \frac{S^2}{\mu_w} + \frac{(1-S)^2}{\mu_o}$ ,  $f(S) = \frac{S^2}{M(S)}$ ,  $S$  = water saturation,  $\phi$  = porosity,  $P$  = pressure in the reservoir,  $\mu_w$ =viscosity of the water phase,  $\mu_o$ = viscosity of the oil phase,  $M(S)$  = total mobility,  $f(S)$  = fractional flow of the water phase,  $V(x, y, t)$  = total fluid velocity.

The initial and boundary conditions for the above partial differential equations areas follows [3, 8]:

$$\begin{cases} \nabla P \cdot n = 0 & \text{on } \Gamma_{\text{imp}} \\ P = P_{\text{in}} & \text{on } \Gamma_{\text{in}} \\ P = P_{\text{out}} & \text{on } \Gamma_{\text{out}} \\ S(x, y, t) = S_{\text{in}} & \text{on } \Gamma_{\text{in}} \\ S(x, y, 0) = S_0 & \text{on } \Omega \end{cases} \quad (2)$$

Where  $n$  = outward unit normal vector defined on  $\Omega$ ,  $P_{\text{in}}$ = injection well pressure,  $P_{\text{out}}$ = production well pressure,  $S_{\text{in}}$ = water saturation in the injection well,  $S_0$ = initial water saturation in the reservoir.

### 3. SIMULATIONS

Flex PDE (PDE Solutions Inc., WA) is software based on the finite element method for solving numerically single or coupled sets of partial differential equations (PDE). Flex PDE can solve steady-state or time-dependent problems. It is able to obtain numerical solutions to PDE by using a problem description language, a finite element numerical modelling facility and graphical output of solutions, without needing to know the complex details of programming or finite element implementation, which makes it is easy to use [3]. Thus, Flex PDE (Student Version 6.19) was used to obtain numerical solutions to the PDE in this project. The parameters for the simulation used in this project were shown in Table 1. The solutions to these equations are beneficial for us to have a better understanding of water flood process and predict such a process. Contour and surface plots of reservoir pressure and water saturation were constructed to visualize change of these two variables with location at a given time. The reservoir pressure and water saturation as a function of injection time at a given location in the reservoir were also predicted.

### 4. RESULTS AND DISCUSSION

The simulation results are shown in Figs. 2-8. As shown in Figs. 2-3 and Figs. 5-6, the reservoir pressure and water saturation gradually decreases with increasing the distance from the injection well. As shown in Fig. 4 and Fig. 7, the reservoir pressure and water saturation gradually increases with increasing injection time at the location of (20 m, 20 m) in the reservoir. As shown in Fig. 8, the water saturation gradually increases with increasing injection time at the production well.

**Table.1.** Parameters for the simulation used in this project.

Parameter	Value	Parameter	Value
Absolute permeability	$K = 1.0 \times 10^{-13} \text{ m}^2$	Initial water saturation	$S_0 = 0$
Porosity	$\phi = 0.206$	Water saturation in injection well	$S_{in} = 1$
Viscosities	$\mu_w = 1.0 \times 10^{-3} \text{ Pa}\cdot\text{s}$ $\mu_o = 4.0 \times 10^{-3} \text{ Pa}\cdot\text{s}$	Wellbore radius	$r_w = 0.11 \text{ m}$
Pressure in water injection well	$P_{in} = 1500 \text{ kPa}$	Unit well dimension	$L = 60 \text{ m}$
Pressure in production well	$P_{out} = 1000 \text{ kPa}$	Simulation time	$t = 1000 \text{ day}$ $= 8.64 \times 10^7 \text{ s}$
Initial pressure in reservoir	$P_0 = 1000 \text{ kPa}$		

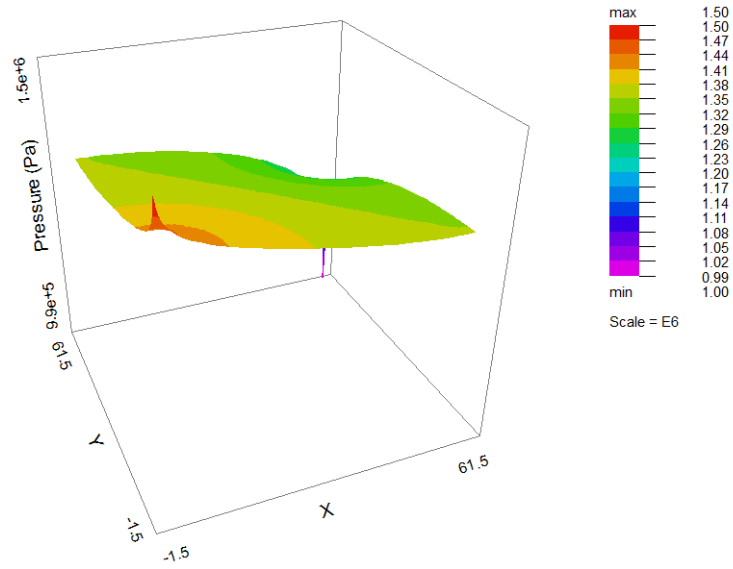


Fig.2.Surface plot of reservoir pressure at 1000 days.

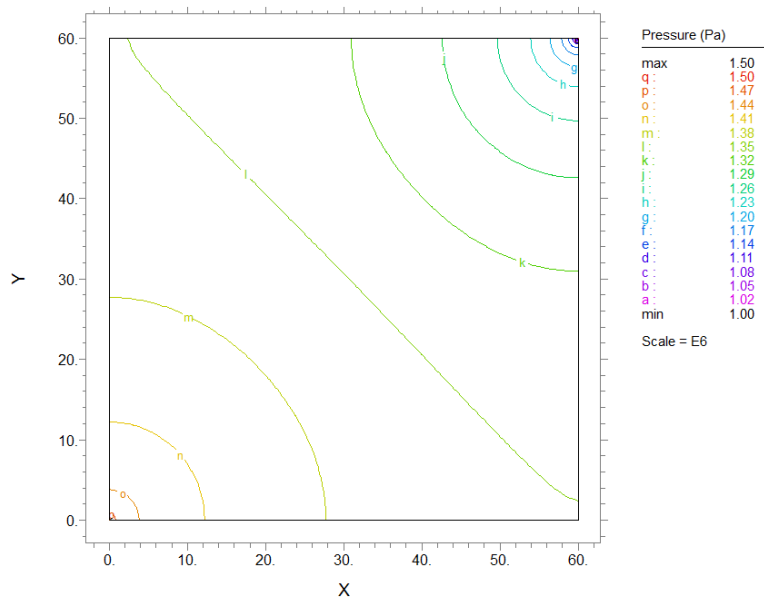


Fig.3.Contour plot of reservoir pressure at 1000 days.

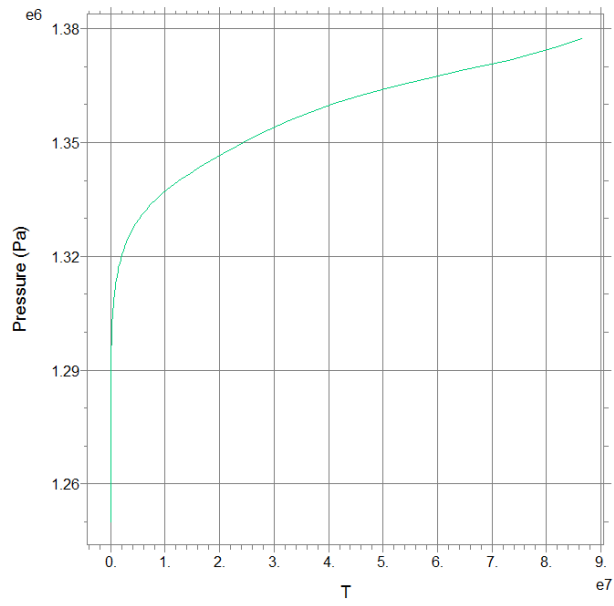


Fig.4. Reservoir pressure at the location of (20 m, 20 m) vs. time (seconds).

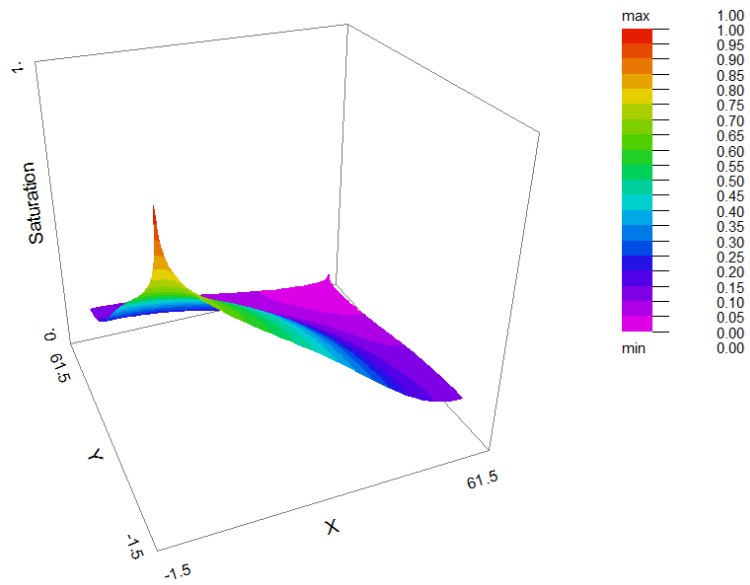


Fig.5. Surface plot of water saturation in the reservoir at 1000 days.

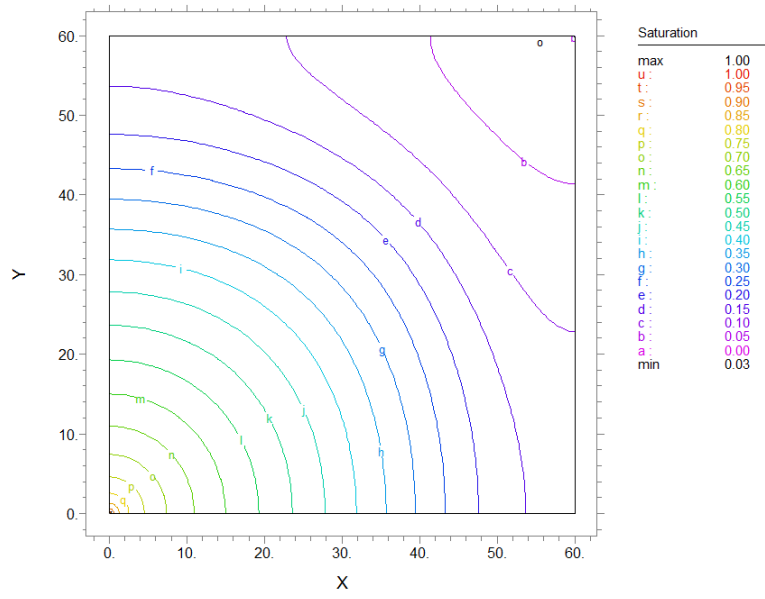


Fig.6. Contour plot of water saturation in the reservoir at 1000 days.

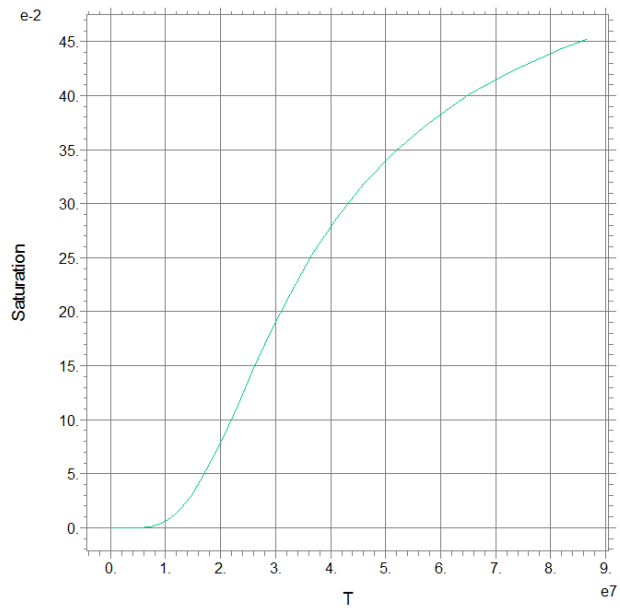
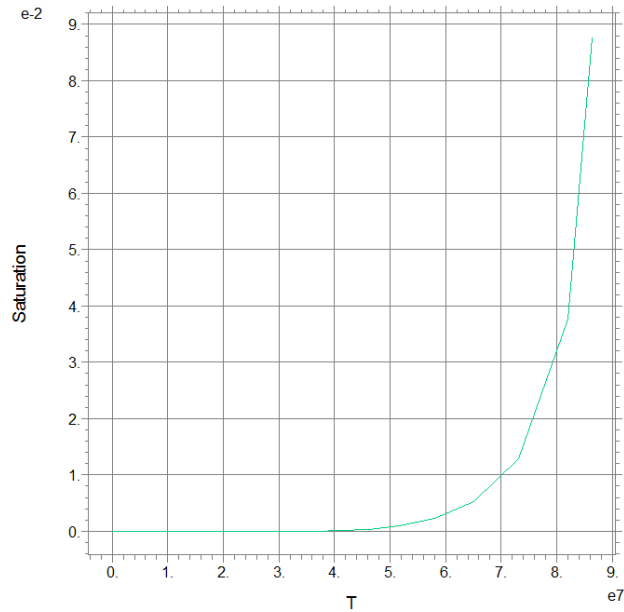


Fig.7. Water saturation in the reservoir at the location of (20 m, 20 m) vs. time (seconds).



**Fig.8.** Water saturation in the production well vs. time (seconds).

## 5. CONCLUSIONS

In this project, the partial differential equations governing a water flood in a unit well pattern were solved numerically and then the numerical solutions were used to construct the contour and surface plots of reservoir pressure and water saturation. The reservoir pressure and water saturation at a given location in the reservoir as a function of injection time were also simulated. The simulation results show that the reservoir pressure and water saturation gradually decreases with increasing the distance from the injection well. In addition the reservoir pressure and water saturation gradually increase with increasing injection time at the location of (20 m, 20 m) in the reservoir. The water saturation gradually increases with increasing injection time at the production well.

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## Nomenclature

$f$  = fractional flow, dimensionless

$K$  = permeability,  $L^2, m^2$

$L$  = length and width of the unit reservoir,  $L, m$

$M$  = mobility, dimensionless

$n$  = outward unit normal vector

$P$  = reservoir fluid pressure,  $M/LT^2, Pa$

$r_w$  = wellbore radius,  $L, m$

$S$  = water saturation, dimensionless



$t$  = time, T, s [day]

$V$  = fluid velocity, L/T, m/s

$x, y$  = distances along the Cartesian directions, L, m

### **Greek Symbols**

$\mu$  = fluid viscosity, M/LT, Pa·s

$\Gamma$  = boundary, dimensionless

$\Omega$  = reservoir domain, L×L, m×m

$\phi$  = porosity, dimensionless

### **Operational**

$\partial$  = partial derivative operator

$\nabla$  = gradient operator

$\cdot$  = dot product

div = divergence operator

### **Subscripts**

0 = initial

imp = impermeability

in = injection well

o = oil phase

out = production well

w = water phase

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