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Forced convection heat transfer of power-law fluids around a confined obstacle cylinders

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Abstract. This paper presents a comprehensive computational work on hydrodynamic , and thermal study of two dimensional incompressible flow of non-Newtonian power law fluid over confined cylinder with four different cross-sectional profiles (square, hexagonal, octagonal, circular) in horizontal channel, the hexagonal and octagonal forms are assumed to be inclined with the angle (α), the research is investigated numerically in steady laminar flow, continuity, momentum and energy equations are solved by using *ANSYS-CFX*. The effects of cross-sectional form, inclination angle (α), Power-law index (n) on drag coefficient and Nusselt number, are studied under these conditions, ($1 \leq Re \leq 40$), ($0 \leq \alpha \leq 48$), ($0.4 \leq n \leq 1.6$), blockage ration $\beta = 0.25$, and $Pr = 50$, This work addresses the combined characteristics of the flow and the heat transfer for power-law fluids around a new cross-sectional shapes, and expands the previous research in this field, the obtained results show the optimal inclination of hexagonal and octagonal cylinder. Moreover, The NDR shows that the hexagonal then the octagonal cross-sectional profiles respectively can be more efficient than circular cylinder.

Keywords: *Power-Law fluids, Steady flow, Polygon cylinder, Drag force, Nusselt number, Heat transfer, Forced convection.*

1. Introduction

The heat and drag force aspects of an obstacle cylinders for different cross-section subjected to fluid flow are the key issue in design and development of such products. The industrial importance of this problem has been due to the common need for information about the fluid flow phenomena such as drag and lift forces and their effects on the rate of heat transfer that are considerable interest in many industrial applications such as drying process, unit of operation… etc. From the point of fluid around obstacle, practical applications such as support structure, off-shore pipe lines, and tall buildings and towers. As a heat transfer mechanism are closely interrelated with the operation performance of electronic cooling, heat exchangers, microchannels, nuclear, and devices of thermal treatment of food staffs. Furthermore, using the different cross-sectional cylinders for increasing the heat transfer while reducing the pressure drop (drag force) is the ultimate goal of several recent works. For the completed application range, experimental and numerical researches have indicated that the thermal performance depends on its flow conditions as well as geometrical parameters (cross-sectional shape, channels …).

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Indeed, many common substances exhibit non-Newtonian flows; these include polymeric, cosmetics and toothpaste, natural substances, biological fluids, foods, etc. Theoretically, Non-Newtonian power-law fluids, or the Oswald relationship is type of generalized Newtonian fluid for which the apparent viscosity characteristic depends with shear rate, the most common type of non-Newtonian power law behavior is shear thickening fluid (dilatants fluid), where the viscosity appears to increase when the shear rate increases, Another familiar example of the opposite, the shear thinning fluid (pseudoplastic), where the viscosity appears to decrease when the shear rate increases, these two behaviors may be represented by the power-law model. Abundant literature is now in hand on different cases pertaining to the flows of Newtonian and non-Newtonian past around unconfined obstacle [1], others around a confined cylinders [2 – 4]. All in all, the main points of studying the power-law fluid flow over a confined cylinders of different cross-section have been dedicated to the investigation of wake, recirculation zone length, drag and lift characteristics, [1] reported the role of Reynolds number and power-law index past unconfined square cylinder they found out that the dilatants fluids decrease the circulation zone length, they reported also results of the pressure and friction coefficient dissipation on unconfined obstacles. Recently, there has been some research in the field of flow past inclined cylinder for the sake of improving the heat transfer rate while minimizing the pressure drop. Rao et al. [5] Numerically investigation of power law fluid flow over elliptical cylinders with different aspect ratios to realize the condition for onset of wage creation and vortex shedding in this limited conditions ($0.2 \leq E \leq 0.5$) and ($0.3 \leq n \leq 1.8$), They also studied the Strouhel number and the time average drag coefficient corresponding to the cessation of steady-flow regime. A Nejat et al. [6] they supported the study of incompressible flow of non-Newtonian power law fluid over a pair of elliptical tandem cylinders confined in channel in this the range of conditions ($0.25 \leq E \leq 2$), ($0.2 \leq n \leq 1.8$), ($1 \leq Pr \leq 100$) ($1 \leq Re \leq 40$), ($1.25 \leq l \leq 20$) l is as a distance between cylinders, the research emphasized on the effect of those parameters on the drag coefficient and heat transfer characteristics of both cylinders, they reported that the NDR number of elliptical cylinder with aspect ratio $E = 0.5$ can be more efficient than circular cylinder in the range of shear-thinning fluids . Alawadhi [7] the unsteady flow regime simulation of laminar forced convection flow past over an in-line elliptical cylinders array with inclination using the finite element method to investigate the improvement of heat transfer, here, the Reynolds number ranges between 125 and 1000, the Prandtl number is fixed at 0.71, The results showed that the inclination of elliptical cylinder improves the heat transfer rate up to 23.8 % but increases the power-law up to 70 %. J. Aboueian-Jahromi et al. [8] studied the effect of inclination angle on the steady flow and heat transfer of power law fluids around a hated inclined square cylinder in confined channel, this study is investigated numerically, the effects of inclination angle , and power law index on the drag and lift coefficients, pumping power and Nusselt number are studied for the conditions $\beta = 1/4$, $Pr = 50$, ($1 \leq Re \leq 40$), ($0 \leq \alpha \leq 45$), ($0.4 \leq n \leq 1.8$). The results showed that the vortex shedding occurs only for $n = 0.4$ and $\alpha = 45$ at $Re = 33$. They also found that the total drag coefficient of the cylinder increases with an increase in the power law index and the maximum total lift coefficient occurs at a = 15.

Based on the mentioned reviews, in this study, our objective is to investigate numerically the hydrodynamic and forced convection heat transfer of Non-Newtonian power-Law fluid flow over a confined cylinder with different cross-sectional shape (square, hexagonal, octagonal, and circular) in a confined channel, and exploring the effects across-sectional shape and inclination angle of a hexagonal and an octagonal forms on drag coefficient and heat transfer rate. In particular, numerical results are presented and discussed for the following conditions: ($0.4 \leq n \leq 1.6$), ($0 \leq \alpha \leq 48$), $\beta = 0.25$ and ($1 \leq Re \leq 40$), $Pr = 50$, However this work addresses the combined characteristics of power low fluid flow and heat transfer. Also larges the previous research in this field, in fact this research improves the hydrodynamics (C_D) and heat (Nu) affects insights, especially around the polygonal cylinders applications, such as design of heat exchangers, and drying systems. Using the power law fluid and optimal cylinder for enhancing the heat transfer rate while reducing the pressure drop.

Problem statement and governing equation:

Consider a heated cylinder which has different cross-sectional forms (square, hexagonal, octagonal and circular) while maintaining an equal cross-sectional area, the cylinder is placed in the cross middle of long two-dimensional channel, the hexagonal and octagonal forms are assumed to be inclined with the angle α as it is shown in the Fig. 1, this work aims to simulate the incompressible flow of power-law fluids around this cylinder. Therefore, due to the numerical considerations, the fluid flow enters the channel with a fully developed velocity profile and constant temperature (T_{in}), and passes the cylinder whose surfaces are maintained at constant temperature (T_w), in this work, the projected height of cylinder in the transverse direction is expressed as: $b = (E \cos \alpha + a \sin \alpha)$, where $(0 \leq \alpha \leq 360/N)$. Furthermore, the inclined polygonal forms have a periodic position and less than ($360/N$) is considered as maximum inclined angle. Moreover, the cross sectional area of a regular polygonal section is given as $A = (N2a/4 \text{ tang}(p/N)) = (NaE/4)$. Where ($N$) is the side number of regular polygon, (a) is a length of each side, and (E) is the twice length of apothem, the ratio of this projected length (b) to the height of the channel (H) defines the blockage ratio β, The distance between the center of the cylinder and the channel inlet, (L_u), is 5 times of the cylinder height (H); the distance between the center of the cylinder and the channel outlet, (L_d), is 20 times of the cylinder height, $L_{ch} = L_u + L_d$ is also the total length of the computational channel.

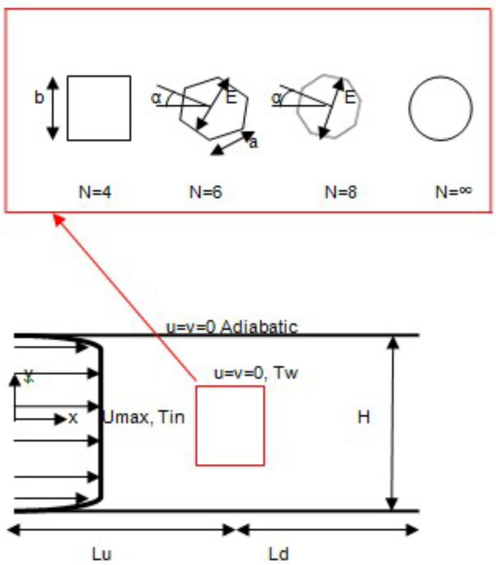

Fig. 1: Schematic of the flow over the inclined and non-inclined obstacles (square, hexagonal, octagonal, circular) cylinder in a plane channel.

Under these considerations, the flow and heat transfer phenomena are governed by continuity, momentum and energy equations. This study is based on the following assumptions: laminar and non-Newtonian flow, steady-state, forced convection, body forces, and constant fluid properties, the governing equations are written in their dimensionless forms as follows:

- Continuity

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \quad (1)$$

- Momentum

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{1}{\mathrm{Re}}\left(\frac{\partial \tau_{xx}^*}{\partial x^*}+\frac{\partial \tau_{yx}^*}{\partial y^*}\right) \quad (2)$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{\partial p^*}{\partial y^*}+\frac{1}{\mathrm{Re}}\left(\frac{\partial \tau_{xy}^*}{\partial x^*}+\frac{\partial \tau_{yy}^*}{\partial y^*}\right) \quad (3)$$

- Energy

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=-\frac{\partial p^*}{\partial y^*}+\frac{1}{Pe}\left(\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right) \quad (4)$$

Where u^* and v^* are the fluid dimensionless velocities in x^* and y^* directions, p^* and T^* are dimensionless pressure and temperature respectively, and *Re* and *Pe* are Reynolds and Peclet numbers. The dimensionless variables are represented as:

$$u^*=u/V_{\max},\ v^*=v/V_{\max},\ x^*=x/d,\ y^*=y/d \quad (5)$$

$$p^*=p/(\rho V_{\max}^2),T^*=(T-T_{in})/(T_w-T_{in}) \quad (6)$$

The behavior of the power-law fluid is represented by the following equation:

$$\tau_{ij}=2\eta\varepsilon_{ij} \quad (7)$$

Where τ_{ij} and ε_{ij} are the rate of deformation and viscous stress tensors respectively. In addition, η which represents the fluid viscosity, is defined (in dimensional form) as follows for power-law fluids:

$$\eta=m\left(\frac{I_2}{2}\right)^{\left(\frac{n-1}{2}\right)} \quad (8)$$

Where n is the power-law index, m is the consistency index and I_2 is the second invariant of the rate of deformation tensor. $n < 1$ represents a shear-thinning fluid, $n = 1$ represents a Newtonian limit and $n > 1$ represents a shear-thickening fluid. In the Cartesian coordinates, I_2 is given by the following equation:

$$\frac{I_2}{2}=2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 \quad (9)$$

Generally, Reynolds number, Prandtl and Grashof number for power-law fluids are computed as follows:

$$\mathrm{Re}=\frac{\rho d^n V_{\max}^{(n-2)}}{m},\mathrm{Pr}=\frac{mc_p}{k}\left(\frac{V_{\max}}{d}\right)^{(n-1)} \quad (10)$$

Where ρ, *cp* and *k* are density, specific heat and thermal conductivity of the fluid, respectively. The Peclet number that is showed in governing equations is obtained from the following equation:

$$Pe = \mathrm{Re} \times \mathrm{Pr} = \frac{\rho c_p d V_{\max}}{k} \quad (12)$$

The boundary conditions used for the flow and heat configuration are:

- At the inlet a fully developed velocity profile for laminar flow of power-law fluids with a constant temperature, this is given by:

$$u^* = U_{\max}(1 - \left|2x^*\beta\right|)^{(\frac{n+1}{n})}, v^* = 0, T^* = 0 \quad (14)$$

- On the surface of the obstacle cylinder: The standard no-slip condition is used and the cylinder is maintained and heated with a constant temperature T_w.

$$u^* = 0, v^* = 0, T^* = 1 \quad (15)$$

- At the channel walls, the usual no-slip condition for flow and adiabatic condition for energy are used.

$$u^* = 0, v^* = 0, Adiabatic \quad (16)$$

- At the outlet Neumann boundary condition for field variables is employed:

$$\frac{\partial v^*}{\partial x^*} = 0, \frac{\partial v^*}{\partial y^*} = 0, \frac{\partial T^*}{\partial y^*} = 0 \quad (17)$$

Overall drag coefficient was mathematically defined as:

$$C_D = \frac{2F_D}{\rho u_{\max}^2 d} \quad (18)$$

F_D is the total drag force on the surface of the cylinder.

The local Nusselt number on the surface of cylinder was evaluated for constant wall temperature as:

$$Nu_l = \frac{hd}{k} = -\frac{\partial \theta}{\partial n_s} \quad (19)$$

Where h and n_s are: the local surface heat transfer coefficient and the normal direction to the cylinder surface. These local values on entire surface were then averaged to obtain the average Nusselt number of circular cylinder.

$$Nu = \frac{1}{S}\int_s Nu_l ds \quad (20)$$

3. Numerical methodology

The conservation equations subjected to the aforementioned boundary conditions are solved using a finite volume based CFD solver ANSYS-CFX version (14.0). ANSYS-CFX software is a high performance, a general purpose fluid dynamics program that is capable of solving diverse and complex three dimensional geometry. This commercial code uses the above equations to describe the principal processes of momentum, mass, and heat transfer; it also combines a specific number of mathematical models such as (power-law, k-e...) that can be used simultaneously with fundamental equations to describe other physical and chemical phenomena such as combustion, turbulence, non-Newtonian flow etc. This present CFD package applies the finite volume method to covert the governing partial differential equations into a system of discrete algebraic equations by discretizing the computational domain into grid mesh. These equations may result in a solution with specified domain boundary conditions. For a transient simulation, an initial condition is also required to numerically close the equations. One of the most important features of CFX is that it uses a coupled solver, which solves the fluid flow and pressure as a single system and faster than the segregated solver up to a certain number of control volumes as it requires fewer iterations to achieve equally converged solutions.

The unstructured trilateral cells of non-uniform grid spacing were generated using the grid package GAMBIT (version 2.4.6). The grids points are distributed in a non-uniform manner wither higher concentration near cylinder Fig. 2 shows the grid used for the case of an octagonal form at $\alpha = 0$.

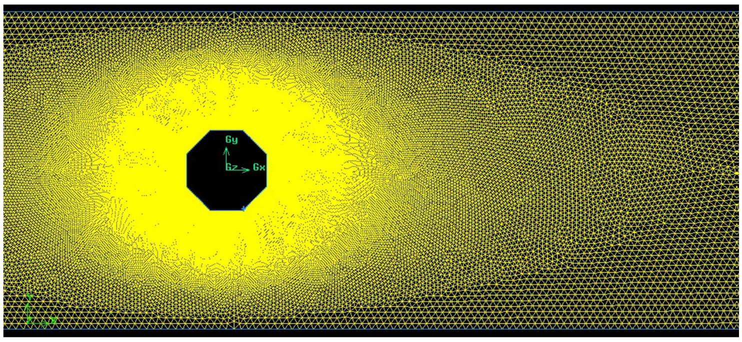

Fig. 2: Representative computational grid unstructured, for an octagonal cylinder, at $\alpha = 0$

3.1. Validation test

In order to test the accuracy of our numerical method, a force convection heat transfer from a heat square cylinder is performed and the average Nusselt number of cylinders in the range of $Re = 1 - 40$ for $n = 1$, $Pr = 50$ and $\beta = 0.25$ is depicted in the fig. 3, an excellent agreement is seen between present results and the results obtained from the literature of Aboueian et al. [8].

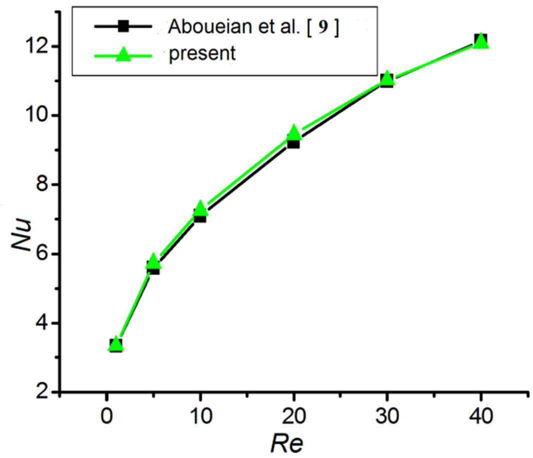

Fig.3: Comparison of Nusselt number for n = 1, for Re = 1 - 40, Pr = 50 and β = 0.25.

4. Results and discussion

As noted earlier, the non-Newtonian flow and heat transfer characteristics of power-law fluid flows passing around different cross-sectional forms of cylinder with different side number (N = 4, 6, 8, and infinity) have been carried out numerically under these conditions Reynolds number ($1 \leq Re \leq 40$), power-Law index ($0.4 \leq n \leq 1,6$), inclination angle α = (0, 12, 24 , 36, 48) for an hexagonal (N = 6) cylinder, (0, 9,18, 27, 36) for an octagonal (N = 8) cylinder, at fixed value of Pr = 50, and β = 0.45. The effects of those parameters on the total drag coefficient and average Nusselt number is the main purpose of this paper.

4.1 Flow patterns

Fig. 4 shows the effects of cross-sectional form and power-law index on the streamlines in the vicinity of cylinder at Re = 40. From this figure it is shown that, a closed steady recirculation region consisting of twin symmetric vortices forms behind the bodies. The size of the recirculation region both along the stream-wise as well as transverse directions decreases with increasing in the power-law index or polygonal side number.

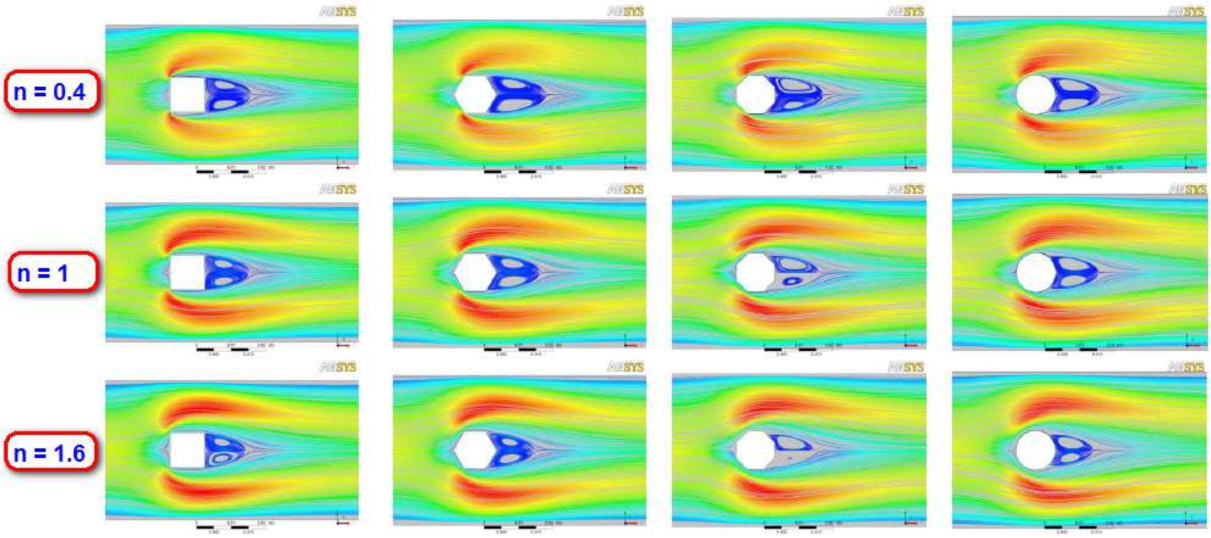

Fig. 4 Streamlines for *Re* = 40, *n* = 0.4, 1, and 1.6, for different cross-sectional form.

Fig. 5 depicts the effect of inclination angle on streamline contours for two forms (hexagonal and octagonal) and for three values of power-law index at *Re* = 30 and *n* = 1. It is presented that the minimum recirculation zone occurred at the angle α = 24 for hexagonal form and α = 18 for and octagonal form.

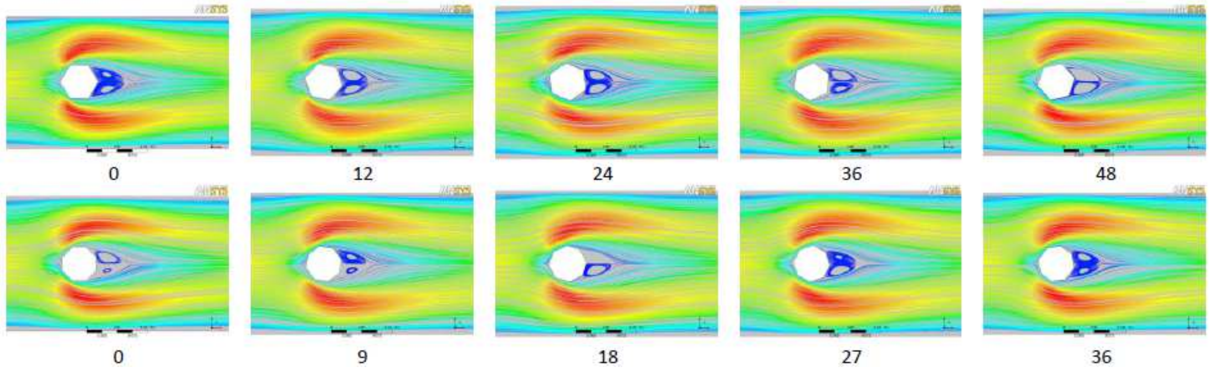

Fig. 5 Streamlines contours around hexagonal and octagonal forms for α = 0 to 48, *Re* = 30 and *n* = 1.

4.2 Total drag coefficient

The total drag coefficient is defined as equation (18), where F_D is the hydrodynamic force exerted by the fluid flow on the obstacle surfaces. Fig. 6 presents the variation of total drag coefficient with power- law index *n* for different cross-sectional forms (square, hexagonal, octagonal and circular) for different value of Reynolds number *Re* = 1 to 40. Fig. 7 depicts the dependence of drag coefficient with inclination angle, power-law index and Reynolds number for hexagonal cylinder. Meanwhile, Fig. 8 shows the effects of inclination angle, power-law index and Reynolds number on total drag coefficient for octagonal cylinder. From those figures, it is shown that the total drag coefficient decreases with decrease in the value of power-law index due to the rheological behavior of viscosity around the obstacle. By other words, a decrease in power-law index leads to lower the apparent viscosity of the fluid beside the cylinder, hence leads to decrease the frictional drag force. Moreover, it is observed that the influence of the power-law index on total drag coefficients gradually diminishes as the Reynolds number is increased from *Re* = 1 to 40. Furthermore, for all values of Reynolds number and power-law index the inclination of hexagonal and octagonal forms shows that, the minimum values of drag coefficient is at the angle α = 24 for an hexagonal form and at the angle α = 18 for an octagonal form.

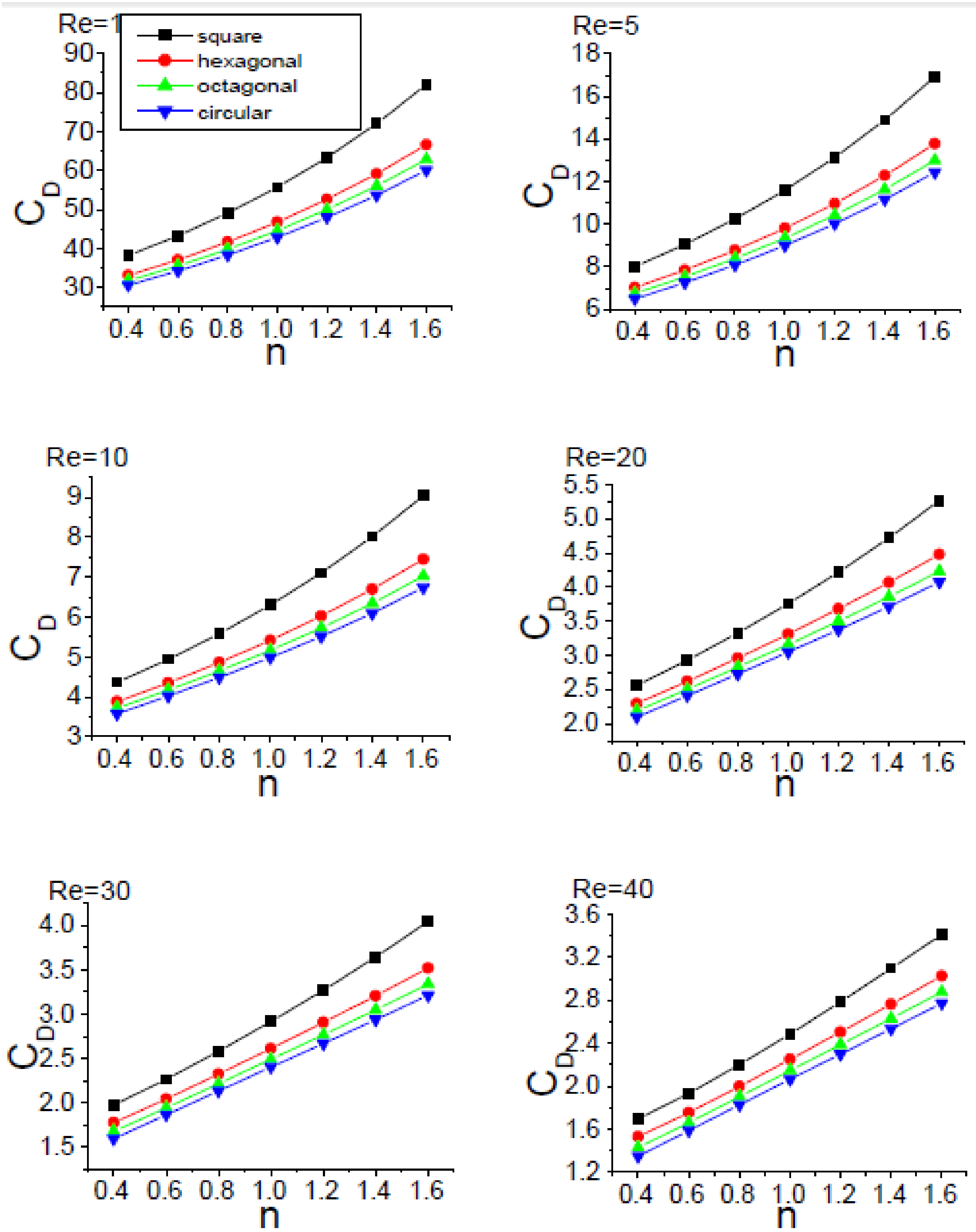

Fig. 6 Effects of cross-section, power law index and Reynolds number on the drag coefficient.

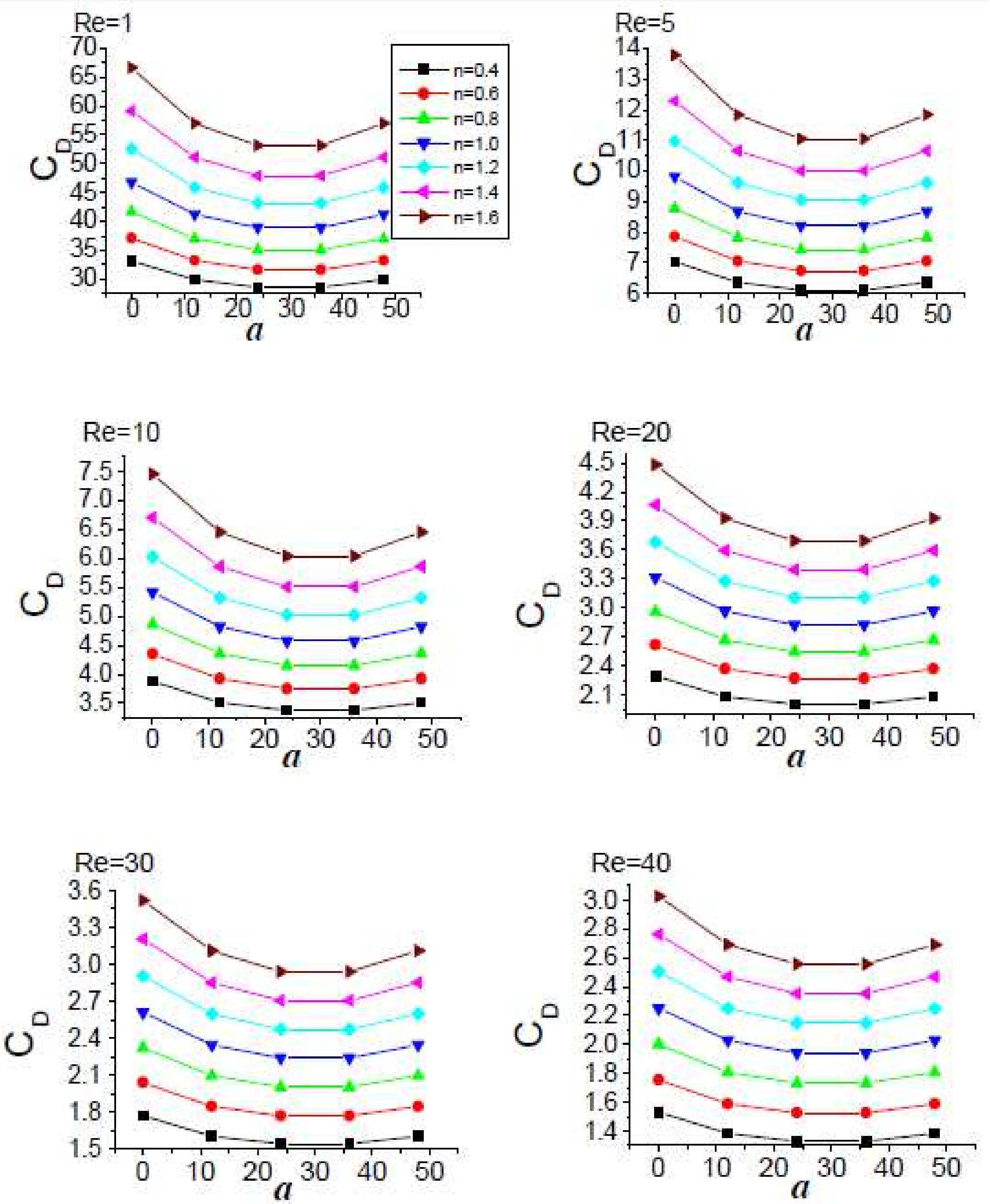

Fig. 7 Effects of inclination angle, power law index and Reynolds number on the drag coefficient for an hexagonal cylinder.

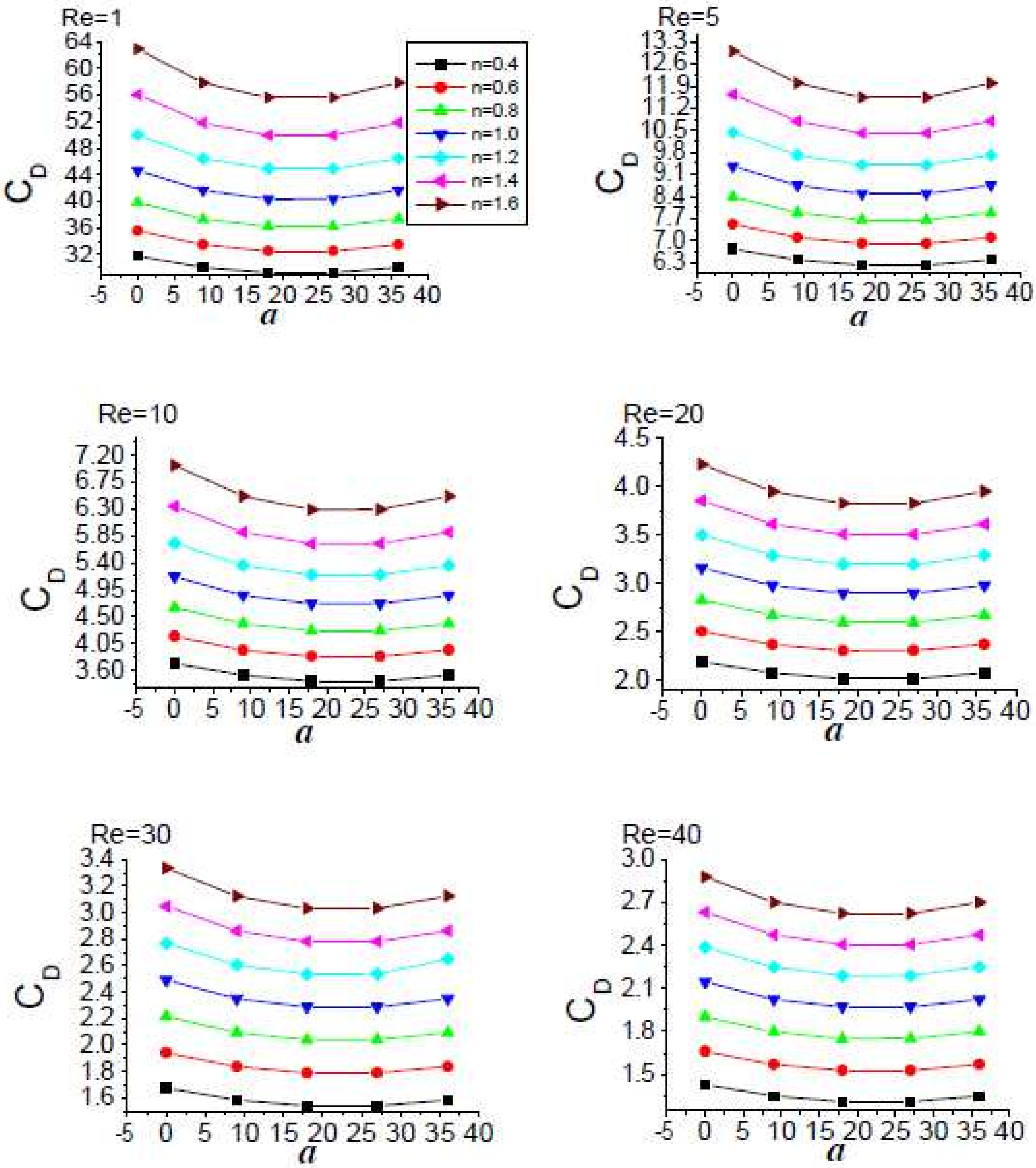

Fig. 7 Effects of inclination angle, power law index and Reynolds number on the drag coefficient for an octagonal cylinder.

4.3 Average Nusselt number

Surely, the changes in the flow field due to rheological value of *n* directly influence on the average Nusselt number of a cylinder, and hence the rates of heat transfer. For that purpose, in this section, we have presented the effects of power-law index, Reynolds number, and geometrical characteristics (*N*, *α*) on the average Nusselt number, at fixed *Pr* number, however, the local Nusselt is defined as equation (19), where *h* is the local heat transfer coefficient. Fig. 8 shows the functional dependence of *Nu* on power-law index and Re at fixed $Pr = 50$, for four different value of *N*. Fig. 9 and 10 depict the variation of Nusselt number with inclination angle for hexagonal and octagonal forms respectively. From those plotted graphs, it is shown that, the Nusselt number increases with increase in Reynolds number due to the progressive thinning of thermal boundary layer. Moreover, increasing degree of shear-thinning (decreasing value of *n*) also enhances the rate of heat transfer and this is because of the lowering of the apparent viscosity of the fluid due to steep gradients near the submerged cylinder. Furthermore, the inclination of hexagonal and octagonal forms show that the maximum heat transfer rate occurs at the angle $\alpha = 24$ for a hexagonal form and at $\alpha = 18$ for an octagonal form.

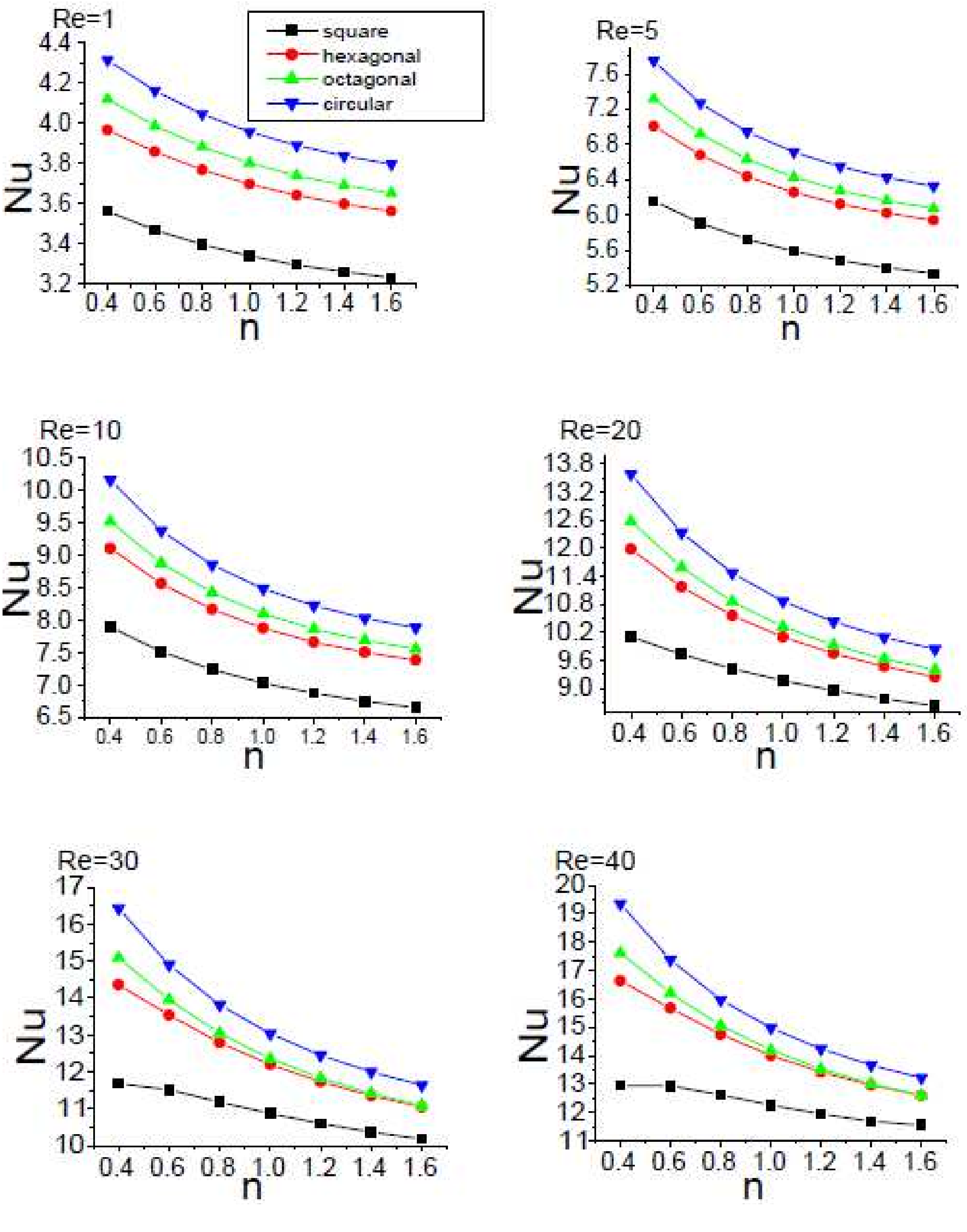

Fig. 8 Effects of cross-section, power-law index and Reynolds number on Average Nusselt number.

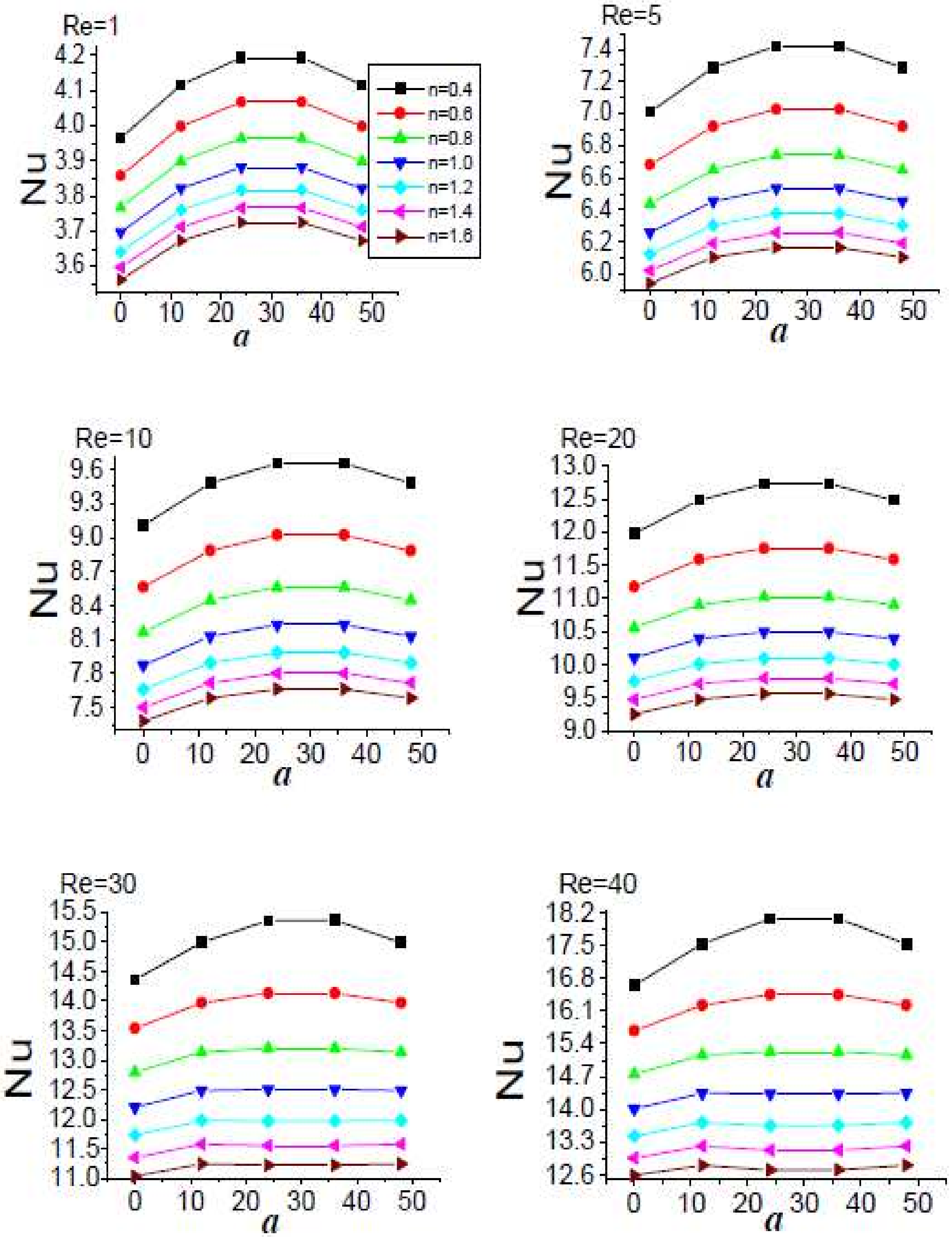

Fig. 9 Effects of inclination angle, power-law index and Reynolds number on Average Nusselt number for a hexagonal from.

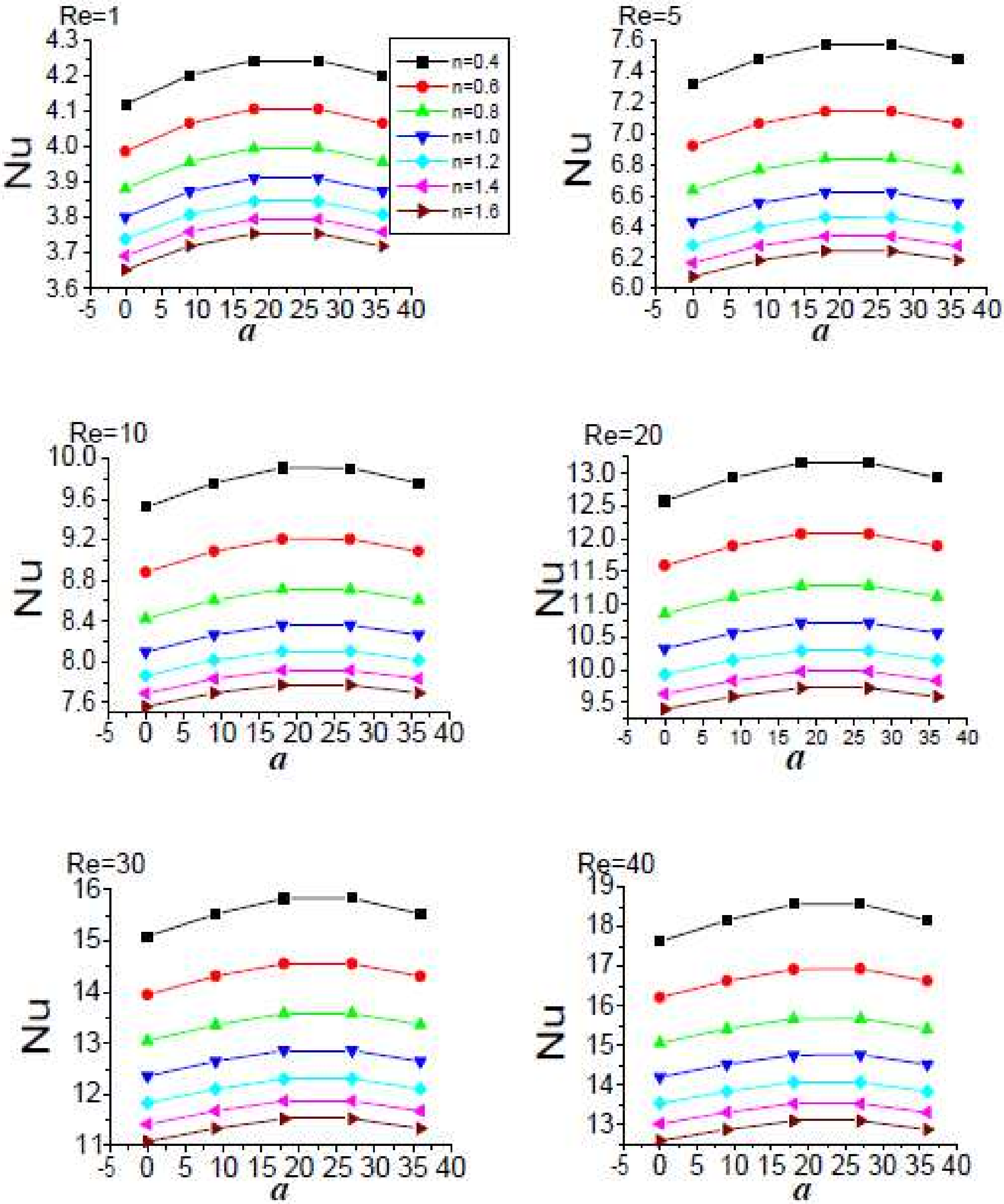

Fig. 10 Effects of inclination angle, power-law index and Reynolds number on Average Nusselt number for an octagonal form.

4.4 Initial parameter of design:

In order to check the efficiency of a cylinder, A. Nejat et al. [6] have proposed a new design parameter, it is introduced as the ratio of Nusselt number to drag coefficient ($NDR = Nu/C_D$). A. Nejat et al. [6] have found that the value of *NDR* of an elliptical form with aspect ratio of 0.5 is larger than circular form only in range of shear thinning fluids. Meanwhile, Fig. 11 depicts that the *NDR* of hexagonal form inclined with 24 degrees then octagonal form inclined with 18 degrees respectively are larger than *NDR* of circular form in the range of all values of power-law indices.

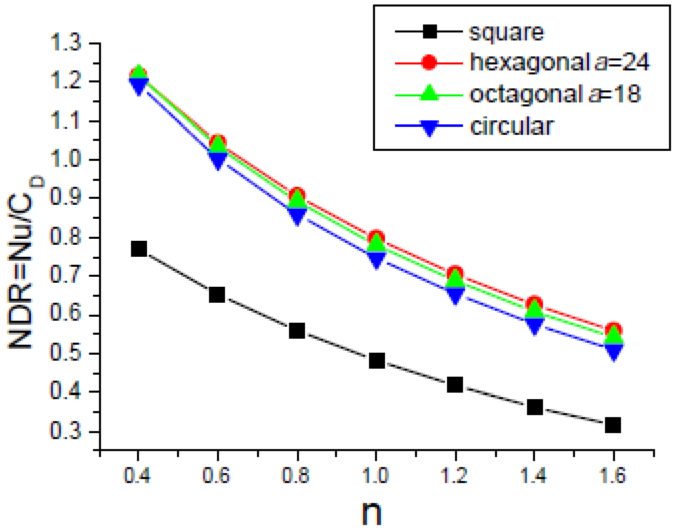

Fig. 11 Variation of *NDR* for the four cross-sectional forms (α = 24, 18) for hexagonal and octagonal forms respectively at *Re* = 5 and *Pr* = 50

5. Conclusions

The effects of the power-law index ($0.4 \le n \le 1.6$), the side number (N= 4, 6, 8, infinity), and the inclination angle ($0 \le \alpha \le 48$) on the flow field and the heat transfer characteristics of a regular confined polygonal cylinder in two dimensional channel ($\beta = 0.25$) at low Reynolds number ($1 \le Re \le 40$) have been studied in detail. The results demonstrate that for all Reynolds number the onset of wake formation was seen to be delayed with an increase in the value of side number (N) and/or with inclined α at its optimal degree, and/or with a decrease in the power-law index. For all cross-sectional forms, the total drag coefficient increases with an increase in power-law index, Moreover, the rate of heat transfer increases with a decrease in the power-law index and/or with an increase in side number. Therefore, using these cross-sectional forms and/or lower power-law indices economically enhance the thermal efficiency of confined cylinder applications, finally the *NDR* shows that the hexagonal then octagonal cylinder with inclination angle α = 24, 18 respectively can be more efficient than circular form for all range of power-law indices.

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