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A problem of spherical cavity in an infinite generalized thermoelastic medium with double porosity subjected to moving heat source

Rajneesh Kumar [a] *, Richa Vohra [b] †

[a] Department of Mathematics, Kurukshetra University, Kurukshetra, Haryana,India
[b] Department of Mathematics & Statistics, H.P.University, Shimla, HP, India

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ABSTRACT

This paper is concerned with the investigation of thermoelastic interactions in an isotropic unbounded medium with spherical cavity due to the presence of moving heat source in context of linear theory of thermoelasticity with one relaxation time [1]. Laplace transform technique has been used to obtain the expressions for radial stress, equilibrated stresses and temperature distribution. A numerical inversion technique has been applied to recover the resulting quantities in the physical domain. The components of stress and temperature distribution are depicted graphically to show the effect of heat source velocity and the relaxation time parameters. Some particular cases are also deduced from the present investigation.

1. Introduction

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Biot [2] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al. [3], where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers.

The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super- saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (so-

*Email : rajneesh_kuk@rediffmail.com
†Email : richavhr88@gmail.com

metimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system .

Aifantis[4-7] introduced a multi-porous system and studied the mechanics of diffusion in solids Wilson and Aifanits [8] presented the theory of consolidation with the double porosity. Khaled et. al [9] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Wilson and Aifantis [8]. Wilson and Aifantis[10]discussed the propagation of acoustics waves in a fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two coexisting degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space.

Nunziato and Cowin [11]developed a nonlinear theory of elastic material with voids. Later, Cowin and Nunziato [12]developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They also considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Nunziato and Cowin have established a theory for the behavior of porous solids in which the skeletal or matrix materials are elastic and the interstices are voids of material.

Beskos and Aifantis [13] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan [14] studied the unified theory of flow and deformation in double porous media. Khalili and Selvadurai [15] presented a fully coupled constitutive model for thermo-hydro –mechanical analysis in elastic media with double porosity structure. Various authors [16-21] investigated some problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity.

Iesan and Quintanilla [22] used the Nunziato-Cowin theory of materials with voids to derive a theory of thermoelastic solids, which have a double porosity structure. This theory is not based on Darcy's law. In contrast with the classical theory of elastic materials with the double porosity, the double porosity structure in the case of equilibrium is influenced by the displacement field.

Youseff [23-25] investigated some problems of infinite body with a cylindrical cavity and spherical cavity in generalized thermoelasticity. Allam et al [26] considered the model of generalized thermoelasticity proposed by Green and Naghdi, to study the electromagneto–thermoelastic interactions in an infinite perfectly conducting body with a spherical cavity. Abd-Alla and Abo-Dahab [27] studied the effect of rotation and initial stress on an infinite generalized magneto-thermoelastic diffusion body with a spherical cavity. Zenkour and Abouelregal [28] studied the effects of phase-lags in a thermoviscoelastic orthotropic continuum with a cylindrical hole and variable thermal conductivity.

The present paper deals with thermoelastic interactions in an infinite double porous thermoelastic body with a spherical cavity subjected to moving heat source in context of Lord-Shulman theory of thermoelasticity. Laplace transform has been applied to find the expressions for the components of stress and temperature distribution. The resulting quantities are obtained in the physical domain by using a numerical inversion technique. Variation of radial stress, equilibrated stresses and temperature distribution against radial distance are depicted graphically to show the effect of heat source velocity and relaxation time parameters. Some particular cases have also been deduced from the present investigation.

2. Governing equations

Following Iesan and Quintanilla [22] and Lord and Shulman [1] ; the constitutive relations and field equations for homogeneous isotropic thermoelastic material with double porosity structure in the absence of body forces and extrinsic equilibrated body forces can be written as :

Constitutive Relations :

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi - \beta\delta_{ij}T \tag{1}$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i} \tag{2}$$

$$\chi_i = b_1\varphi_{,i} + \gamma\psi_{,i} \tag{3}$$

Equation of motion :

$$\mu\nabla^2 u_i + (\lambda+\mu)\, u_{j,ji} + b\varphi_{,i} + d\psi_{,i} - \beta T_{,i} = \rho\ddot{u}_i, \tag{4}$$

Equilibrated Stress Equations of motion :

$$\alpha\nabla^2\varphi + b_1\nabla^2\psi - bu_{r,r} - \alpha_1\varphi - \alpha_3\psi + \gamma_1 T = \kappa_1\ddot{\varphi}, \tag{5}$$

$$b_1\nabla^2\varphi + \gamma\nabla^2\psi - du_{r,r} - \alpha_3\varphi - \alpha_2\psi + \gamma_2 T = \kappa_2\ddot{\psi}, \tag{6}$$

Equation of heat conduction :

$$\left(1 + \tau_0\frac{\partial}{\partial t}\right)\left(\beta T_0\dot{u}_{j,j} + \gamma_1 T_0\dot{\varphi} + \gamma_2 T_0\dot{\psi} + \rho C^*\dot{T} - Q\right) = K^*\nabla^2 T \tag{7}$$

where λ and μ are Lame's constants, ρ is the mass density ; $\beta = (3\lambda + 2\mu)\,\alpha_t$; α_t is the linear thermal expansion ; C^* is the specific heat at constant strain, u_i is the displacement components ; t_{ij} is the stress tensor ; κ_1 and κ_2 are coefficients of equilibrated inertia ; σ_i is the components of the equilibrated stress vector associated to pores ; χ_i is the components of the equilibrated stress vector associated to fissures ; φ is the volume fraction field corresponding to pores ψ and is the volume fraction field corresponding to fissures ; K^* is the coefficient of thermal conductivity ; Q is the heat source ; τ_0 is the thermal relaxation time, κ_1 and κ_2 are coefficients of equilibrated inertia $b, d,\ b_1, \gamma, \gamma_1, \gamma_2$ and are constitutive coefficients ; δ_{ij} is the Kronecker's delta ; T is the temperature change measured form the absolute temperature T_0 ($T_0 \neq 0$) ; a superposed dot represents differentiation with respect to time variable t.

We take the moving heat source as :

$$Q = Q_0 H(r-R)\frac{\delta(r-vt)}{r} \tag{8}$$

where $H(*)$ is the Heaviside unit step function, Q_0 is the heat source strength and v is its velocity.

3. Formulation of the problem

We consider a perfectly conducting thermoelastic infinite body with double porosity having spherical cavity occupying the region $R \leq r < \infty$ of an isotropic homogeneous medium.The spherical polar coordinates (r, ϑ, ϕ) are taken for any representative point of the body at time t and the origin of the coordinate system is at the centre of the spherical cavity. All the variables considered will be functions of the radial distance r and the time t. The initial conditions are given by

$$u = 0 = \dot{u}, \quad \varphi = 0 = \dot{\varphi}, \ \ \psi = 0 = \dot{\psi}, \quad T = 0 = \dot{T} \quad \text{at } t = 0 \tag{9}$$

Due to spherical symmetry, the displacements components are of the form

$$u_r = u(r,t), \quad u_\vartheta = u_\phi = 0 \tag{10}$$

The components of stress tensor for a spherical symmetric system are

$$t_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e + b\varphi + d\psi - \beta T \tag{11}$$

$$t_{\vartheta\vartheta} = 2\mu\frac{u}{r} + \lambda e + b\varphi + d\psi - \beta T \tag{12}$$

$$t_{r\vartheta} = t_{r\phi} = t_{\vartheta\phi} = 0 \tag{13}$$

$$\sigma_r = \alpha \frac{\partial \varphi}{\partial r} + b_1 \frac{\partial \psi}{\partial r} \tag{14}$$

$$\chi_r = b_1 \frac{\partial \varphi}{\partial r} + \gamma \frac{\partial \psi}{\partial r} \tag{15}$$

where

$$e = e_{rr} + e_{\vartheta\vartheta} + e_{\phi\phi} = \frac{\partial u}{\partial r} + \frac{2u}{r}, \tag{16}$$

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\vartheta\vartheta} = e_{\phi\phi} = \frac{u}{r}, \quad e_{r\vartheta} = e_{r\phi} = e_{\vartheta\phi} = 0 \tag{17}$$

We introduce the non-dimensional quantities as :

$$\begin{aligned} &r' = \tfrac{\omega_1}{c_1} r, \quad u' = \tfrac{\omega_1}{c_1} u \,, t'_{ij} = \tfrac{t_{ij}}{\beta T_0}, \varphi' = \tfrac{\kappa_1 {\omega_1}^2}{\alpha_1} \varphi, \quad \psi' = \tfrac{\kappa_1 {\omega_1}^2}{\alpha_1} \\ &T' = \tfrac{T}{T_0}, \ t' = \omega_1 t, \sigma'_i = \left(\tfrac{c_1}{\alpha \omega_1}\right) \sigma_i, \ \chi'_i = \left(\tfrac{c_1}{\alpha \omega_1}\right) \chi_i, \tau'_0 = \omega_1 \tau_0, Q'_0 = \tfrac{c_1 Q_0}{K^* \omega_1 T_0} \end{aligned}$$

where $c_1^2 = \frac{\lambda + 2\mu}{\rho}, \omega_1 = \frac{\rho C^* c_1^2}{K^*}$

Making use of dimensionless quantities given by (17) on Eqs. (4)-(7) and with the aid of Eqs. (8) and (16) yield (dropping primes for convenience)

$$\frac{\partial e}{\partial r} + a_1 \frac{\partial \varphi}{\partial r} + a_2 \frac{\partial \psi}{\partial r} - a_3 \frac{\partial T}{\partial r} = \frac{\partial^2 u}{\partial t^2} \tag{18}$$

$$a_4 \nabla^2 \varphi + a_5 \nabla^2 \psi - a_6 e - a_7 \varphi - a_8 \psi + a_9 T = \frac{\partial^2 \varphi}{\partial t^2} \tag{19}$$

$$a_{10} \nabla^2 \varphi + a_{11} \nabla^2 \psi - a_{12} e - a_{13} \varphi - a_{14} \psi + a_{15} T = \frac{\partial^2 \psi}{\partial t^2} \tag{20}$$

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right) \left(a_{16} \frac{\partial e}{\partial t} + a_{17} \frac{\partial \varphi}{\partial t} + a_{18} \frac{\partial \psi}{\partial t} + \frac{\partial T}{\partial t} - \frac{Q_0 H(r-R) \delta(r - \nu t)}{r}\right) = \nabla^2 T \tag{21}$$

where

$$\begin{aligned} &a_1 = \tfrac{b \alpha_1}{\rho c_1^2 \kappa_1^2 \omega_1^2}, a_2 = \tfrac{d \alpha_1}{\rho c_1^2 \kappa_1^2 \omega_1^2}, a_3 = \tfrac{\beta T_0}{\rho c_1^2}, a_4 = \tfrac{\alpha}{\kappa_1 c_1^2}, a_5 = \tfrac{b_1}{\kappa_1 c_1^2}, a_6 = \tfrac{b}{\alpha_1}, a_7 = \tfrac{\alpha_1}{\kappa_1 \omega_1^2}, \\ &a_8 = \tfrac{\alpha_3}{\kappa_1 \omega_1^2}, a_9 = \tfrac{\gamma_1 T_0}{\alpha_1}, a_{10} = \tfrac{b_1}{\kappa_2 c_1^2}, a_{11} = \tfrac{\gamma}{\kappa_2 c_1^2}, a_{12} = \tfrac{d \kappa_1}{\kappa_2 \alpha_1}, a_{13} = \tfrac{\alpha_3}{\kappa_2 \omega_1^2}, a_{14} = \tfrac{\alpha_2}{\kappa_2 \omega_1^2}, \\ &a_{15} = \tfrac{\gamma_2 T_0 \kappa_1}{\alpha_1 \kappa_2}, a_{16} = \tfrac{\beta c_1^2}{\rho C^*}, a_{17} = \tfrac{\gamma_1 \alpha_1 c_1^2}{K^* \kappa_1 \omega_1^3}, a_{18} = \tfrac{\gamma_2 \alpha_1 c_1^2}{K^* \kappa_1 \omega_1^3} \end{aligned}$$

4. Solution in the Laplace transform domain

Applying the Laplace transform defined by

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t)\, e^{-st} dt \tag{22}$$

on the Eqs. (18)-(21), after some simplifications, we obtain

$$\left[\nabla^8 + B_1\nabla^6 + B_2\nabla^4 + B_3\nabla^2 + B_4\right] \bar{e} = \frac{f_1}{r} e^{-(s/\nu)r} \tag{23}$$

$$\left[\nabla^8 + B_1\nabla^6 + B_2\nabla^4 + B_3\nabla^2 + B_4\right] \bar{\varphi} = \frac{f_2}{r} e^{-(s/\nu)r} \tag{24}$$

$$\left[\nabla^8 + B_1\nabla^6 + B_2\nabla^4 + B_3\nabla^2 + B_4\right] \bar{\psi} = \frac{f_3}{r} e^{-(s/\nu)r} \tag{25}$$

$$\left[\nabla^8 + B_1\nabla^6 + B_2\nabla^4 + B_3\nabla^2 + B_4\right] \bar{T} = \frac{f_4}{r} e^{-(s/\nu)r} \tag{26}$$

B_i, f_i ; $i = 1, 2, 3, 4$ are given in the appendix.

Therefore, the solutions of the Eqs. (23)-(26), which is bounded at infinity, are given by

$$\bar{e} = \frac{A_1}{r} e^{-m_1 r} + \frac{A_2}{r} e^{-m_2 r} + \frac{A_3}{r} e^{-m_3 r} + \frac{A_4}{r} e^{-m_4 r} + \frac{D_1}{r} e^{-(s/v)r} \tag{27}$$

$$\bar{\varphi} = g_{11}\frac{A_1}{r} e^{-m_1 r} + g_{12}\frac{A_2}{r} e^{-m_2 r} + g_{13}\frac{A_3}{r} e^{-m_3 r} + g_{14}\frac{A_4}{r} e^{-m_4 r} + \frac{D_2}{r} e^{-(s/v)r} \tag{28}$$

$$\bar{\psi} = g_{21}\frac{A_1}{r} e^{-m_1 r} + g_{22}\frac{A_2}{r} e^{-m_2 r} + g_{23}\frac{A_3}{r} e^{-m_3 r} + g_{24}\frac{A_4}{r} e^{-m_4 r} + \frac{D_3}{r} e^{-(s/v)r} \tag{29}$$

$$\bar{T} = g_{31}\frac{A_1}{r} e^{-m_1 r} + g_{32}\frac{A_2}{r} e^{-m_2 r} + g_{33}\frac{A_3}{r} e^{-m_3 r} + g_{34}\frac{A_4}{r} e^{-m_4 r} + \frac{D_4}{r} e^{-(s/v)r} \tag{30}$$

$g_{1i}, g_{2i}, g_{3i}, g_{4i}$ are given in the appendix.

$$D_i = \frac{f_i v^8}{s^8 + B_1 s^6 \nu^2 + B_2 s^4 \nu^4 + B_3 s^2 \nu^6 + B_4 \nu^8} \quad ; i = 1, 2, 3, 4 \tag{31}$$

Substituting Eqs. (27) into Eq.(16), we obtain

$$\bar{u} = -\frac{D_1}{r^2}\left(\frac{v^2}{s^2} + r\frac{v}{s}\right)e^{-(s/v)r} - \frac{1}{r^2}\sum_{i=1}^{4}\frac{A_i}{m_i^2}\left(1 + rm_i\right)e^{-m_i r}$$

Making use of Eqs.(27)-(30),(32) in Eqs.(11),(14) ,(15) and with the help of Eqs.(17) and (22), we obtain the corresponding expressions for radial stress and equilibrated stresses as

$$\bar{t}_{rr}(r,s) = G_5(r)e^{-(s/v)r} + \sum_{i=1}^{4}\left(-\frac{p_1}{r}\left(m_i^2 + \frac{2m_i}{r} + \frac{2}{r^2}\right) + p_2 + p_3 g_{1i} + p_4 g_{2i} - g_{3i}\right)A_i(s)e^{-m_i r} \tag{32}$$

$$\bar{\sigma}_r(r,s) = -G_6(r)e^{-(s/v)r} - \sum_{i=1}^{4}\left(p_5 g_{1i} + p_6 g_{2i}\right)\left(\frac{m_i r + 1}{r^2}\right)A_i(s)e^{-m_i r} \tag{33}$$

$$\bar{\chi}_r(r,s) = -G_7(r)e^{-(s/v)r} - \sum_{i=1}^{4}\left(p_6 g_{1i} + p_7 g_{2i}\right)\left(\frac{m_i r + 1}{r^2}\right)A_i(s)e^{-m_i r} \tag{34}$$

where

$$p_1 = \tfrac{2\mu}{\beta T_0},\;\; p_2 = \tfrac{\lambda}{\beta T_0},\;\; p_3 = \tfrac{b\alpha_1}{\beta T_0\kappa_1\omega_1^2},\;\; p_4 = \tfrac{d\alpha_1}{\beta T_0\kappa_1\omega_1^2},\; p_5 = \tfrac{\alpha_1}{\kappa_1\omega_1^2},\;\; p_6 = \tfrac{b_1\alpha_1}{\alpha\kappa_1\omega_1^2},\;\; p_7 = \tfrac{\gamma\alpha_1}{\alpha\kappa_1\omega_1^2},$$
$$G_5 = -\tfrac{1}{r}\left(p_1 D_1\left(\tfrac{2v^2}{s^2r^2} + \tfrac{2v}{sr} + 1\right) + p_2 D_1 + p_3 D_2 + p_4 D_3 - D_4\right),$$
$$G_6 = (p_5 D_2 + p_6 D_3)\left(\tfrac{s}{vr} + \tfrac{1}{r^2}\right),\;\; G_7 = (p_6 D_2 + p_7 D_3)\left(\tfrac{s}{vr} + \tfrac{1}{r^2}\right)$$

5. Boundary conditions

We consider that the bounding plane ($r = R$) of the cavity is traction free and subjected to thermal shock as follows :

$$t_{rr}(R,t) = 0,\;\; \sigma_r(R,t) = 0, \chi_r(R,t) = 0,\;\; T(R,t) = T_0 H(t) \tag{35}$$

After applying Laplace transform on Eq.(36), we get

$$\bar{t}_{rr}(R,s) = 0,\;\; \bar{\sigma}_r(R,s) = 0,\;\; \bar{\chi}_r(R,s) = 0,\;\; \bar{T}(s,t) = \frac{T_0}{s} = F_1 \text{ (say)} \tag{36}$$

Substituting the values of $\bar{t}_{rr}, \bar{\sigma}_r, \bar{\chi}_r$ and $\bar{T}$ from Eqs. (30), (33)-(35) in the boundary conditions (37) yield the corresponding expressions for radial stress, equilibrated stresses and temperature distribution as

$$\bar{t}_{rr}(r,s) = \frac{1}{\Gamma}\left(H_{11}\Gamma_1 \exp(-m_1 r) + H_{12}\Gamma_2 \exp(-m_2 r) + H_{13}\Gamma_3 \exp(-m_3 r) + H_{14}\Gamma_4 \exp(-m_4 r)\right) \tag{37}$$

$$\bar{\sigma}_{r}(r,s) = \frac{1}{\Gamma}\left(H_{21}\Gamma_1 \exp(-m_1 r) + H_{22}\Gamma_2 \exp(-m_2 r) + H_{23}\Gamma_3 \exp(-m_3 r) + H_{24}\Gamma_4 \exp(-m_4 r)\right) \tag{38}$$

$$\bar{\chi}_{r}(r,s) = \frac{1}{\Gamma}\left(H_{31}\Gamma_1 \exp(-m_1 r) + H_{32}\Gamma_2 \exp(-m_2 r) + H_{33}\Gamma_3 \exp(-m_3 r) + H_{34}\Gamma_4 \exp(-m_4 r)\right) \tag{39}$$

$$\bar{T}(r,s) = \frac{1}{\Gamma}\left(H_{41}\Gamma_1 \exp(-m_1 r) + H_{42}\Gamma_2 \exp(-m_2 r) + H_{43}\Gamma_3 \exp(-m_3 r) + H_{44}\Gamma_4 \exp(-m_4 r)\right) \tag{40}$$

where

$$\Gamma = \begin{vmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{vmatrix} \tag{41}$$

$\Gamma_i \ (i = 1\ , 2, 3, 4)$ are obtained by replacing i^{th} column of (41) with $[\ F_2 \ \ F_3 \ \ F_4 \ \ (F_5 + F_1)\ ]^T$

$$\begin{aligned}
&H_{1i} = \tfrac{p_1}{m_i^2}\left(\tfrac{m_i^2}{R} + \tfrac{2m_i}{R^2} + \tfrac{2}{R^3}\right) + p_2 + p_3 g_{1i} + p_4 g_{2i} - g_{3i},\\
&H_{2i} = \left(\tfrac{m_i R+1}{R^2}\right)(p_5 g_{1i} + p_6 g_{2i}),\ H_{3i} = \left(\tfrac{m_i R+1}{R^2}\right)(p_6 g_{1i} + p_7 g_{2i}), H_{4i} = g_{3i},\\
&F_2 = \tfrac{1}{R}\left(p_1 D_1\left(\tfrac{2v^2}{s^2R^2} + \tfrac{2v}{sR} + 1\right) + p_2 D_1 + p_3 D_2 + p_4 D_3 - D_4\right)e^{-(s/v)R},\\
&F_3 = (p_5 D_2 + p_6 D_3)\left(\tfrac{s}{vR} + \tfrac{1}{R^2}\right)e^{-(s/v)R},\\
&F_4 = (p_6 D_2 + p_7 D_3)\left(\tfrac{s}{vR} + \tfrac{1}{R^2}\right)e^{-(s/v)R}, F_5 = \tfrac{D_4}{R}e^{-(s/v)R}
\end{aligned}$$

6. Particular cases

Case 6.1 If $\tau_0 = 0$, in Eqs. (38)-(41) yield the corresponding expressions for an infinite thermoelastic double porous body with a spherical cavity in the context of coupled theory of thermoelasticity.

Case 6.2 If $b_1 = \alpha_3 = \gamma = \alpha_2 = \gamma_2 = d \to 0$ in Eqs.(38)-(41), we obtain the corresponding expressions for an infinite thermoelastic single porous body with a spherical cavity.

7. Inversion of the Laplace domain

In order to invert the Laplace transform, we adopt a numerical inversion method based on a Fourier series expansion [29]

By this method the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by

$$f(t) = \frac{e^{\eta t}}{t_1}\left[\frac{1}{2}\bar{f}(\eta) + \mathrm{Re}\sum_{k=1}^{N}\bar{f}\left(\eta + \frac{ik\pi}{t_1}\right)\exp\left(\frac{ik\pi t}{t_1}\right)\right], \quad 0 < t_1 < 2t$$

where N is sufficiently large integer representing the number of terms in the truncated Fourier series, chosen such that

$$f(t) = \exp(\eta t) Re\left[\bar{f}\left(\eta + \frac{iN\pi}{t_1}\right)\exp\left(\frac{iN\pi t}{t_1}\right)\right] \leq \varepsilon_1$$

where is a prescribed small positive number that corresponds to the degree of accuracy required. The parameter is a positive free parameter that must be greater than the real part of all the singularities of .The optimal choice of was obtained to the criterion described in [29].

8. Numerical results and discussion

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief and Saleh [30] as,

$\lambda = 7.76 \times 10^{10} Nm^{-2}$, $C^* = 3.831 \times 10^3 m^2 s^{-2} K^{-1}, \mu = 3.86 \times 10^{10} Nm^{-2}$,

$K^* = 3.86 \times 10^3 Ns^{-1}K^{-1}$, $T_0 = 293\ K, \alpha_t = 1.78 \times 10^{-5} K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$

The double porous parameters are taken as,

$\alpha_2 = 2.4 \times 10^{10} Nm^{-2}, \alpha_3 = 2.5 \times 10^{10} Nm^{-2}, \gamma = 1.1 \times 10^{-5} N, \alpha = 1.3 \times 10^{-5}\ N$

$\gamma_1 = 0.16 \times 10^5 Nm^{-2}, b_1 = 0.12 \times 10^{-5}\ N, d = 0.1 \times 10^{10} Nm^{-2}$

$\gamma_2 = 0.219 \times 10^5 Nm^{-2}, \kappa_1 = 0.1456 \times 10^{-12} Nm^{-2}s^2, b = 0.9 \times 10^{10} Nm^{-2}$

$\alpha_1 = 2.3 \times 10^{10}\ Nm^{-2}$, $\kappa_2 = 0.1546 \times 10^{-12} Nm^{-2}s^2$

The other non-dimensional parameters are taken as

$Q_0 = 5.0$, $t = 0.2$, $R = 1.0$, $\tau_0 = 0.1$

The software MATLAB has been used to find the values of radial stress t_{rr}, equilibrated stresses σ_r, χ_r and temperature distribution T. The variations of these values with respect to radial distance r have been shown in figures (1)-(8). In figs.1-4, effect of thermal relaxation time is shown graphically. In all these figures, solid line and small dashed line correspond to Lord-Shulman(LS) theory of thermoelasticity for to coupled theory (CT)of thermoelasticity respectively. Also, the effect of heat source velocity is depicted graphically in figs. 5-8 for different values of heat source velocity parameters $\nu = 0.2$, 0.4 and 0.6.

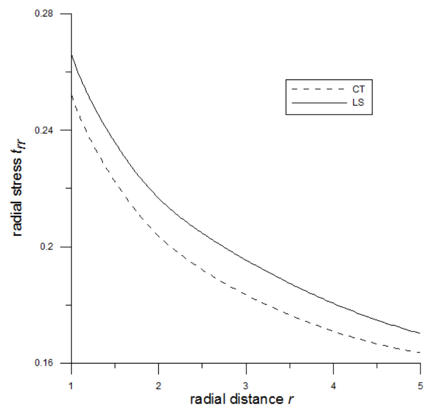

Fig. 1 – Variation of radial stress t_{rr} w.r.t. radial radial distance r.

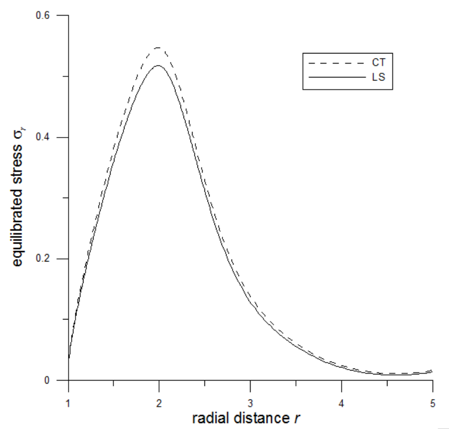

Fig. 2 – Variation of equilibrated stress σ_r w.r.t. radial distance r.

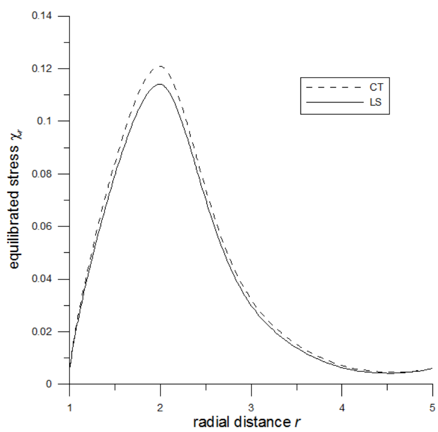

Fig. 3 – Variation of equilibrated stress χ_r w.r.t. radial radial distance r.

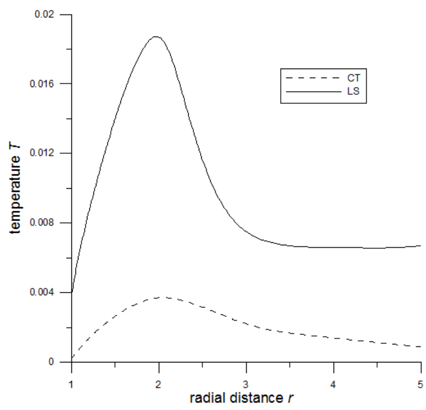

Fig. 4 – Variation of temperature distribution T w.r.t. radial distance r.

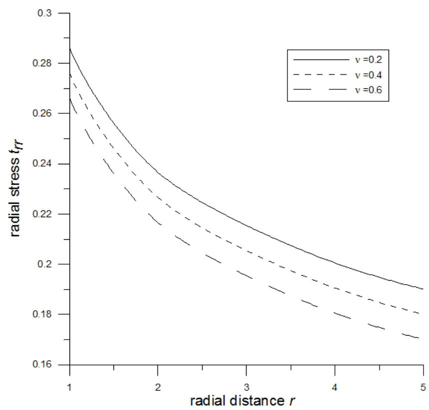

Fig. 5 – Variation of radial stress t_{rr} w.r.t. radial radial distance r.

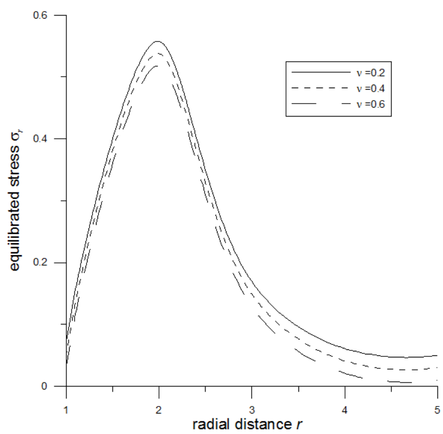

Fig. 6 – Variation of equilibrated stress σ_r w.r.t. radial distance r.

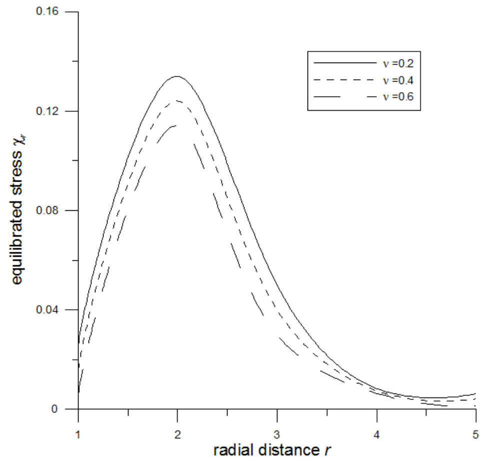

Fig. 7 – Variation of equilibrated stress χ_r w.r.t. radial r.

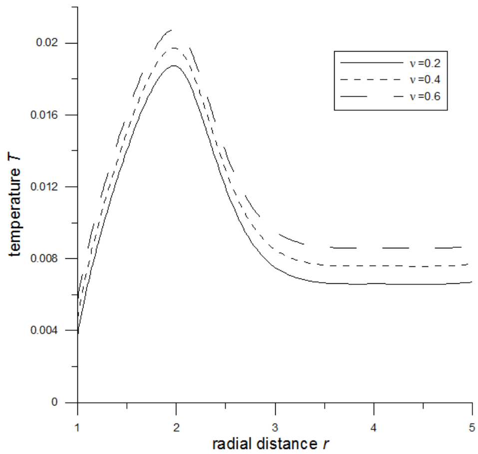

Fig. 8 – Variation of temperature distribution T w.r.t. radial distance r.

Fig.1 shows that radial stress t_{rr} is maximum at the boundary surface of the spherical cavity and it decreases monotonically with increase in radial distance r. Also, it is found that the magnitude values of t_{rr} increases due to relaxation time parameter. The values of t_{rr} are more for LS theory in comparison to CT theory of thermoelasticity . From figs.2 and 3 , it is clear that equilibrated stresses σ_r and χ_r increases for $1 \leq r \leq 2$ and then decreases onwards as increases. The magnitude values of σ_r and χ_r decreases due to relaxation time. It is evident that the values of σ_r and χ_r are more for CT theory as compared to the values for LS theory of thermoelasticity. Fig.4 depicts that the values of temperature distribution T increase monotonically for $1 \leq r \leq 2$, decrease monotonically for $2 \leq r \leq 3$ and then decrease very slowly and steadily with the increase in the value of radial distance r. It is also found that relaxation time parameter increases the values of T, the magnitude value of are more incase of LS theory than that of CT theory of thermoelasticity.

Fig.5 represents that radial stress t_{rr} decreases monotonically with increase in radial distance r. Also, it is found that the magnitude values of t_{rr} decrease with the increase in the values of heat source velocity ν. Figs.6 and 7 shows that equilibrated stresses σ_r and χ_r increase for $1 \leq r \leq 2$ and then start decreasing as $r > 2$. It is also clear that the magnitude values of σ_r and χ_r decrease as the value of heat source velocity ν increases. Fig.8 depicts that the values of temperature distribution T increase monotonically for $1 \leq r \leq 2$, decrease monotonically for $2 \leq r \leq 3$ and then become almost stationary as $r > 3$. Also. it is found that as the velocity of heat source increases, the magnitude values of temperature distribution T increases also increases.

9. Concluding remarks

In this work, we have studied the problem of infinite thermoelastic medium with double porosity having spherical cavity in context of Lord-Shulman theory of thermoelasticity with one relaxation time subjected to moving heat source. Effect of thermal relaxation time and heat source velocity parameters are shown graphically on radial stress, equilibrated stresses and temperature distribution. All the field quantities are observed to be very sensitive towards the heat source velocity parameter. From figures, it is concluded that the magnitude values of radial stress and equilibrated stresses decrease with increase in the values of heat source velocity while a reverse trend is noticed in case of temperature distribution. The thermal relaxation time parameter has also a considerable effect on the all the physical quantities. The relaxation time parameter has both the increasing as well as decreasing effect on these quantities which shows that it is very important to take into account the relaxation time parameter.

This type of study is useful due to its application in geophysics and rock mechanics. The results obtained in this investigation should prove to be beneficial for the researchers working on the theory of thermoelasticity with double porosity structure. The introduction of double porous parameter to the thermoelastic medium represents a more realistic model for further studies.

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Appendix

$$\begin{aligned}
&a_{19} = s(1+\tau_0 s),\ a_{20} = s(1+\tau_0 s)a_{16},\ a_{21} = s(1+\tau_0 s)a_{17},\ a_{22} = s(1+\tau_0 s)a_{18},\\
&n_1 = -\left(a_7 + s^2\right), n_2 = -\left(a_{14} + s^2\right), r_1 = a_5 a_{10} - a_4 a_{11},\\
&r_2 = a_4(a_{11}a_{19} - n_2) - a_{11}n_1 - a_7 a_{10} - a_5(a_{10}a_{19} + a_{13}),\\
&r_3 = n_1(a_{11}a_{19} - n_2) + a_4(n_2 a_{19} - a_{15}a_{22}) + a_5(a_{13}a_{19} + a_{15}a_{21}) +\\
&\qquad a_7(a_{10}a_{19} + a_{13}) + a_9(a_{10}a_{22} - a_{11}a_{21}),\\
&r_4 = n_1(n_2 a_{19} - a_{15}a_{22}) - a_8(a_{13}a_{23} + a_{15}a_{21}) - a_9(a_{13}a_{22} + n_2 a_{21}), r_5 = a_6 a_{11} - a_5 a_{12},\\
&r_6 = -a_6(a_{11}a_{19} - n_2) + a_7 a_{12} + a_5(a_{19}a_{12} + a_{15}a_{20}) - a_9 a_{11} a_{20},
\end{aligned}$$

$$\begin{aligned}
&r_7 = -a_6(n_2 a_{19} - a_{15}a_{22}) - a_7(a_{12}a_{19} + a_{15}a_{20}) - a_8(a_{12}a_{22} + n_2 a_{20}),\\
&r_8 = a_6 a_{10} - a_4 a_{12}, r_9 = -a_6(a_{13} + a_{10}a_{19}) - n_1 a_{12} + a_4(a_{12}a_{19} + a_{15}a_{20}),\\
&r_{10} = a_9(a_{13}a_{20} - a_{12}a_{21}) + n_1(a_{12}a_{19} + a_{15}a_{20}) + a_6\left(a_{13}a_{19} + a_{15}a_{21}\right),\\
&r_{11} = a_{20}(a_4 a_{11} - a_5 a_{10}), r_{12} = a_6(a_{11}a_{21} - a_{10}a_{22}) + a_{20}(n_1 a_{11} + a_7 a_{10})\\
&+a_4(a_{12}a_{22} + n_2 a_{20}) + a_5(a_{13}a_{20} - a_{12}a_{21}),\\
&r_{13} = a_7(a_{12}a_{21} - a_{13}a_{20}) + a_6(a_{13}a_{22} + n_2 a_{21}) + n_1(a_{12}a_{22} + n_2 a_{20}),
\end{aligned}$$

$$\begin{aligned}
&B_1 = (r_2 - s^2 r_1)/r_1, B_2 = (r_3 - s^2 r_2 - a_1 r_5 + a_2 r_8 + a_3 r_{11})/r_1,\\
&B_3 = (r_4 - s^2 r_3 - a_1 r_6 + a_2 r_9 + a_3 r_{12})/r_1, B_4 = (-s^2 r_4 - a_1 r_7 + a_2 r_{10} + a_3 r_{13})/r_1,
\end{aligned}$$

$$\begin{aligned}
&f_1 = \zeta\left(r_1 s^6 + r_2 s^4 v^2 + r_3 s^2 v^4 + r_4 v^6\right)/\nu^6,\quad f_2 = -\zeta\left(r_5 s^4 + r_6 s^2 v^2 + r_7 v^4\right)/\nu^4\\
&f_3 = \zeta\left(r_8 s^4 + r_9 s^2 v^2 + r_{10} v^4\right)/\nu^6,\quad f_4 = \zeta\left(r_{11} s^4 + r_{12} s^2 v^2 + r_{13} v^4\right)/\nu^4,\ \zeta = Q_0 H(r-R)/v\,,
\end{aligned}$$

$$\begin{aligned}
&g_{1i} = -\{r_5 m_i^4 + r_6 m_i^2 + r_7\}\,/\,\{r_1 m_i^6 + r_2 m_i^4 + r_4\},\\
&g_{2i} = \{r_8 m_i^4 + r_9 m_i^2 + r_{10}\}\,/\,\{r_1 m_i^6 + r_2 m_i^4 + r_3 m_i^2 + r_4\},\\
&g_{3i} = -\{r_{11} m_i^4 + r_{12} m_i^2 + r_{13}\}\,/\,\{r_1 m_i^6 + r_2 m_i^4 + r_3 m_i^2 + r_4\};\qquad i = 1,2,3,4
\end{aligned}$$